Derivation

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Abstract

Descartes was right about the method and wrong about the starting point. Submit everything to radical doubt and what survives is not *I think* but what the cogito already presupposes: *there is*. Beginning there, this paper derives — not postulates — a non-perspectival ground. Recognition is shown to carry an irreducible duality of apprehending and apprehended; the mutual determination of unity and multiplicity is established as necessity; self-reference with return forces the threshold of universal computation; the metric completion of its configuration space, with the requirement that consequence chains can close, forces a carrier compact, simply connected, and without boundary — uniquely S^2 in the minimal dimension. Each step is *found*, not invented, and the chain admits machine-checked verification: a warrant whose validity conditions hold independent of the state in which the derivation was found. What it grounds is a depth ordering that is non-perspectival without being a view from nowhere; what it cannot close is the adequacy of the formalization to the insight — checkable, yet not surveyable — which is where the derivation meets, and expresses, the non-closure it describes.

The Ground

A depth ordering of perspectives, if it is to be non-arbitrary, cannot rest solely on any one perspective's self-description; it requires a criterion invariant across perspectives. The strongest form such a criterion could take is a non-perspectival ground — not another perspective but the structural condition under which all perspectives arise. The deepest prior attempt to establish such a ground in Western philosophy remains Descartes's. His method was sound: submit everything to radical doubt, identify what survives, build only from what is indubitable. In the *Discourse on Method* and *Meditations*, he systematically stripped away sensory knowledge, demonstrative reasoning, and all prior belief until only the cogito remained. The methodology was the right one. What can be improved is its application — specifically, the choice of starting point that emerges from the doubt.

The cogito requires a knowing subject to verify its own reliability. Descartes secures this reliability through clear and distinct perception, guaranteed by a non-deceiving God whose existence is itself demonstrated via clear and distinct perception. The circle was identified immediately — Arnauld named it in the Fourth Objections — and has never been resolved (Descartes, 1637/2006; 1641/1984). The fix is not to abandon the Cartesian method but to improve its application. The cogito begins too late — it begins with a subject and then cannot escape the subject's epistemic situation. Begin earlier. Begin with what even the cogito presupposes: *there is*. Something is. The radical skeptic cannot deny this because the denial itself is something that is. This is more indubitable than "I think" because it requires no thinker.

The derivation proceeds in five linked movements: (1) the pre-differentiated unity of apprehending and apprehended; (2) the structural invariant that multiplicity requires unity, established as logical necessity rather than aesthetic preference; (3) geometric structure — which is what distinction and closure *are* when made precise — and the minimal geometric carrier determined under explicit constraints; (4) a demonstration that existing formal tools — trigonometric, categorical, computational — are sufficient to make the full formalization increasingly tractable; (5) the recovery of contemplative content and the rescue of theological content as structural consequence rather than cultural artifact. Of these, the first three are structural arguments, the fourth is a demonstration of capability, and the fifth is partially conditional on the chain's completion: the recovery of contemplative content draws on independent empirical warrant, while the rescue of theological content depends on the derivation — a distinction that will matter throughout.

From Existence to Knowing

"There is" does not stand alone. Recognition — even silent, even pre-linguistic — is always already both apprehending as activity and apprehended as content. This is not asserted but demonstrated by self-application: any attempt to reduce the structure instantiates it. To deny the duality is to perform an act of denial (apprehending) directed at a content (apprehended). To claim bare existence without recognition is to recognize bare existence. The structure cannot be eliminated by any operation that is itself an instance of the structure. This is not paradox — it runs productively rather than destructively. The Liar says "this statement is false" and loops without resolution. The self-application here says "any act performed on this structure presupposes this structure" and *terminates* — not in contradiction but in confirmation. The primitive is not decomposable because decomposition is an instance of it.[1]

The move from "I think" to "there is" has been attempted before, though the present derivation arrives at it from different considerations. Heidegger's *es gibt* — "it gives," or "there is" — insists on the pre-subjective

[1]This structure is cognate with Husserl's intentionality — no act of consciousness without an object, no object without an act — and with Fichte's self-positing of the I as necessarily also a positing of the Not-I. The argument here arrives at the same structure without importing either framework.

character of presence in a way cognate with the starting point here, though Heidegger's concern is with the history of Being rather than formal derivation (Heidegger, 1927/1962). Levinas's *il y a* begins similarly but moves immediately to the ethical relation (Levinas, 1947/1978). Certain Buddhist logicians — Dignāga and Dharmakīrti — begin with the structure of *pramāṇa* (valid cognition) in ways that parallel the move from "there is" to the co-given structure of apprehending and apprehended. Nishitani begins from the same place via the Kyoto School and connects to the emptiness of emptiness as structural rather than nihilistic (Nishitani, 1961/1982). The present derivation shares the starting point with these traditions while proceeding differently: the irreducibility of "there is" is established not by phenomenological evidence but by self-application — any operation performed on the starting point presupposes it — and the aim is formal derivation rather than phenomenological description or ethical grounding. The convergence across independent starting points is noted as confirmatory.

Duality is not added to the primitive. It is the primitive's structure. And the implication is immediate: individuality already contains plurality, and plurality necessitates individuality. The method is not to impose this but to find it — yielding to the terms rather than attempting determination.

Descartes separated the knower from the known and then could not reconnect them. The separation was the error. The post-Cartesian tradition inherited the problem and attempted to solve it from the wrong side — reconstructing unity after the split rather than recognizing it before.

Invariants

Recognition, carefully attended to, reveals that meaning carries implicit perspectival frames — and necessarily more than one. These are not imposed from outside; they are discovered by yielding to what recognition already contains.

Plotinus gives us: *the multiple cannot exist without the simple*. This is his central structural argument, developed throughout the *Enneads* (especially III.8 and V.1; see V.3.12, V.4.1, V.6.3; Plotinus, 1991). If there is multiplicity — and there is, recognition itself already containing both cognizing and cognized — then there must be a unity from which the multiplicity arises.

If multiple, then simple. Multiple. Therefore simple. But the implication runs both ways. What does unity mean within a necessary context like this? As a preliminary intuition: perhaps unity composes in some parallel way to how recognition decomposes.

1. **Local unity**: starting from unity, we can equally prove multiplicity. The two are mutually determinative and necessitating. Each element of a multiplicity must always already have unity for multiplicity to have meaning, and the individuated elements must always already have the unity dimension in order to be grouped. Each individual element relies upon its unity-type relations and its multiplicity-type relations just as seen with the individual and the plural. Defining an item by its unitary-dimension relations and its multiplicity-dimension relations is another way of saying the item is identical to itself. This is not a jump. This is the definition of identity.[2]

2. **Formal unity** follows: any multiplicity considered *as a multiplicity* is already one collection, one field. The collection presupposes a unifying condition. Without the other side of the boundary given

[2]This mutual determination has a precise categorical correlate: in category theory, an object is individuated by its identity morphism and defined by its morphisms to other objects — neither exists without the other. Lawvere's formalization of the unity of opposites exhibits this structure explicitly (Lawvere, 1996). The derivation arrives at it from attention rather than axiomatics; the convergence is treated as structural evidence, not coincidence. That the same structure is independently recoverable from phenomenological attention and from categorical formalization — two radically different generating methods — is itself a case of one structure surviving translation between radically different methods of generation without loss.

by the topology of multiplicity, unity would be meaningless — even a single-element unity requires multiplicity for its individuation to have meaning and for its identity to be defined.

3. **Ground unity**: the mutual determination of unity and multiplicity demonstrated at local and formal levels is reflexively stable — applying the analysis to itself yields confirmation rather than contradiction, since the analysis of mutual determination is itself an instance of mutual determination.[3]

Reflexive stability raises the regress question: if the analysis of mutual determination is itself an instance of mutual determination, what stops infinite iteration? The answer is that regress and recursion are not the same. Regress is self-similar but sterile — each level restates what the last one said. Recursion produces: each application yields formally characterizable structure the prior step did not contain. The difference is generativity. "There is" is a fixed point under self-application — any attempt to decompose it instantiates it — which means the recursion is well-founded: the primitive is already the base case, and each step terminates by adding structure rather than merely restating the relation. The chain produces. It does not loop.

There must therefore be a ground condition of unity, and given mutual determinacy, a corresponding ground condition of multiplicity.

The temptation to reify emerges from attending to terminology with phenomenologically transformative attention like this. What has actually been proven? Given the invariant necessities for meaning, we have proven that rigorous epistemology can be actualized — without being able to say anything so rigorous about the particular instantiations we examined. We end up with an actualized epistemology and the recognition that everything else — all particulars, including unity and multiplicity — have a relative existence and participate in knowing only to a marginal degree compared to the actualization of epistemology itself. They are constituted instead by mutually interdependent origination. This is the structural content that Nāgārjuna's *pratītyasamutpāda* names: no term has independent existence; each arises in mutual dependence. The convergence is not borrowed but derived.

This is not a unique terminological case. We followed Plotinus's example discernments — local, formal, ground — for ease, speed, and because those who have already walked a path literally make it easier for all who come after. We could have used many other groups of terms. The particular instantiations, or even groups of instantiations, are not the important part. What Thoreau calls rebirth into the Father Tongue — the language earned through discipline rather than inherited through convention (Thoreau, 1854) — is the method of attention itself, not any particular vocabulary it yields.

If particular terms — including unity and multiplicity — have only relative existence, and rigorous certainty attaches to the actualized epistemology rather than to any instantiation within it, then how do we continue to build? This is Descartes's question, now asked from firmer ground: what can be constructed with certainty when the construction materials themselves have dependent origination?

The answer: work not with particular terms but with the invariant constraints discoverable across them.

The Minimal Geometric Carrier

The primitive — "there is" — now carries the invariant constraints established above in *Invariants*. What ground do those constraints determine? We require a ground that is:

1. **Non-arbitrary** — not chosen from among alternatives but determined by the constraints themselves.

[3] Lawvere's fixed-point theorem establishes that any structure with sufficient self-referential capacity has fixed points — elements mapped to themselves under the structure's own operations. Reflexive stability is the derivation's instance of this: mutual determination, applied to itself, returns itself. The connection to universal computation follows directly: any adequate formal representation of this structure must accommodate self-reference, which is precisely what characterizes the equivalence class of universal computation.

2. **Minimal** — containing no more structure than required (the Plotinian invariant: any additional complexity would itself be multiplicity demanding a simpler source).
3. **Structurally generative** — capable of producing, through necessary consequence, more than it contains.

The derivation chain itself has self-referential structure — *Invariants* established this directly. The primitive is a fixed point under self-application. Mutual determination is reflexively stable. The recursion is well-founded, with each step yielding formally characterizable structure the prior step did not contain explicitly. Any formal system adequate to represent this chain must handle self-reference with return. In the Chomsky hierarchy (Chomsky, 1956), that capacity first appears at Type 0: below it (Types 1, 2, 3), self-reference with return is not a structural property of the system. Above it, the capacity is present but with unentailed additions. By the minimality constraint already established: Type 0. That contemplative phases independently exhibit this structure — semantic self-evaluation as an empirical instance of self-reference with return — functions as independent confirmation, the same structure reached by a different route; but the derivation does not depend on it.

The ground is what Type 0 expressibility requires as its structural condition: a system capable of self-reference with return, operating on a configuration space that admits recurrent orbits. This is universal computation characterized structurally rather than by a particular formalism. The ground has a configuration space; the geometric derivation proceeds from that space.

That every independent formalization of "effective procedure" converges to the same boundary — Turing machines (Turing, 1936), lambda calculus (Church, 1936), recursive functions (Kleene, 1936), cellular automata, production systems — is then evidence that this threshold is a structural feature rather than an artifact of any particular formalism. The Church-Turing thesis is not a theorem (it cannot be proved, because "effectively computable" is an informal notion), but ninety years without a counterexample and the proved equivalence of every alternative formalism constitute the strongest possible evidence short of proof.

At this threshold, Kleene's recursion theorem (Kleene, 1938) establishes that for any total computable function f there exists a program e such that $\varphi_e = \varphi_{f(e)}$ — self-reference with return is a *theorem*, not an additional assumption. The closure present in the primitive — recognition returning to itself, mutual determination completing — is therefore a provable property of any system at the universal computation threshold. Universal computation does not merely generate; it closes.

Computation Is Already Geometric

Rule 110 — eight input patterns, eight output bits, one-dimensional lattice — is among the simplest structures that achieves universal computation (Cook, 2004). It is not an abstract machine to which geometry is subsequently added — it is geometry that happens to compute: state evolution on a spatial substrate, from the beginning.

This is the general case. Any computation traces a trajectory through its configuration space — the space of all possible states. That space has natural metric structure: two configurations are neighbors if they differ by a single cell; edit distance is the measure. The transition function defines a flow. The topology is not imposed. It is what "next configuration" means when taken seriously as a structural relationship.

The discrete character of this space is not borrowed from the Turing machine formalism — it follows from the mutual determination established above. The mutual determination of unity and multiplicity requires distinguishable elements: to be one configuration is to not be another. Distinguishable means discrete. Each configuration is this or that; nothing intermediate.

The discrete configurations are tiles. The transition rules are adjacency rules — which tile can follow which. The derivation's closure requirement (consequence chains must be able to return) constrains which sequences can converge and to what. Completion fills in whatever geometry these constraints force.

This is the key move, and it is not chosen: the continuous geometry is what the tiles tile out. Every configuration that convergent sequences approach is a state the ground cannot exclude — the Cauchy completion of the edit-distance metric space is already there, forced by what convergence requires. Closure already requires boundedness: a space extending without bound admits sequences that escape arbitrarily far, leaving always further to flee, making return impossible. Complete and totally bounded is compact. The completion inherits Kleene's recurrence structure: fixed-point programs trace recurrent orbits, and these persist as compact invariant sets — not merely fixed points but attractors, regions the dynamics cannot leave once entered. This is why closure is available at every point in the space but not guaranteed along every trajectory: it is a possibility the structure affords, not an outcome it enforces — a contingent achievement in a space where the attractor structure exists but need not be reached.

Within this completed space, a consequence chain that returns to its origin is a loop. A loop is contractible if it can be continuously deformed to a point — the chain can be unwound without remainder. A non-contractible loop threads a topological obstruction, a hole, representing a chain that cannot close without passing through inaccessible structure. The requirement that all consequence chains can close is the requirement that no such obstructions exist: every loop contracts. This is $\pi_1 = 0$ — simple connectivity — derived from closure, not imported. It characterizes the ground's topology: structural absence of obstruction, not enforcement of return.

Different tiles with different adjacency rules would tile out different geometry. These tiles, these rules, this closure constraint — they tile out a space that is compact, simply connected, without boundary. The shape wasn't picked. It was found.

Glass fishing floats from Japanese vessels wash ashore on the Pacific coast — hollow spheres that have crossed an ocean intact. They survive because of their geometry: closed, compact, no edge to catch or break, curvature distributing force evenly across the surface. No point bears more load than any other. The sphere has no preferred position. This isotropy is not incidental to its survival but constitutive of it — the geometry is the reason, not a description of the reason.

The constraints now intersect: compact, simply connected, without boundary. Dimension 1 admits no compact simply connected manifold without boundary — the circle S^1 fails because $\pi_1(S^1) = \mathbb{Z}$, meaning loops that wind once around cannot contract to a point, and the constraints are unsatisfiable. Dimension 2 is where they first admit a solution, and the solution is unique: S^2. Higher-dimensional spheres satisfy the same topological constraints — S^3 is the unique compact simply connected 3-manifold (Perelman, 2003) — but each additional dimension introduces degrees of freedom not entailed by any step in the derivation. The ground, by the minimality already established, contains no more structure than the constraints require. An unentailed degree of freedom is excess structure. Dimension 2 is therefore selected not by preference but by the intersection of two derived constraints: it is the first dimension where the requirements are satisfiable, and any higher dimension would introduce structure the derivation has not generated. Since each prior step in the chain introduced structure under compulsion — forced by what preceded it — unentailed structure in the carrier would break the derivation's own logical character. Dimension 2 is not merely minimal; it is the only dimension the derivation's logic licenses. That this carrier already generates the geometric, physical, and epistemic content explored in subsequent sections is a result to be checked, not an assumption built in.

Power Tools

What are the two highest-leverage formalisms applicable to our indexed frame? One resolves S^2 to computational precision — every point, every curvature value, every invariant quantity made explicit. The other abstracts compositional structure to the limit of what formalization can reach — tracking what is preserved across any domain translation whatsoever.

Trigonometry and category theory.

Trigonometry: The Geometry of Closure

Trigonometry characterizes S^2 with full precision. The 2-sphere of radius r centered at the origin is defined by:

$$x^2 + y^2 + z^2 = r^2$$

It is parameterized by two angles — colatitude $\theta \in [0, \pi]$ and azimuth $\phi \in [0, 2\pi)$:

$$x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi, \quad z = r \cos\theta$$

One constraint (equidistance from center), one number (radius), two parameters — and the surface closes on itself in every direction. The Gauss-Bonnet theorem makes the curvature structure precise. For any compact surface M without boundary, with Gaussian curvature K:

$$\int_M K \, dA = 2\pi\chi(M)$$

The total curvature of any closed surface is fixed by its topology alone. $\chi(M)$ is the Euler characteristic — a topological invariant unchanged by deformations that preserve the surface's topological type. For the 2-sphere, $\chi(S^2) = 2$, giving:

$$\int_{S^2} K \, dA = 4\pi$$

No amount of stretching, compressing, or bending (without tearing or gluing) changes this number. The curvature is a property of the structure itself, not of any particular coordinate system or embedding.

Note that Gauss-Bonnet fixes *total* curvature, not constant curvature. That the standard 2-sphere has *constant* positive curvature is a separate fact: the round metric is the maximally symmetric metric on S^2, the unique metric (up to scaling) admitting $SO(3)$ as its isometry group.

Category Theory: The Logic of Composition

Category theory provides the language for tracking what is preserved as the derivation moves across domains.

A category consists of objects, morphisms (structure-preserving maps between objects), composition of morphisms, and identity morphisms, satisfying associativity and identity laws. A functor maps one category to another while preserving this compositional structure. The power of category theory is that it describes the *pattern of relationship* independent of what is being related — the same compositional logic applies whether the objects are topological spaces, differential manifolds, physical theories, or logical propositions.

For the present work, category theory formalizes what the preceding sections have been doing implicitly: each move from recognition to mutual determination to reflexive stability to universal computation asked "what is preserved in this translation?" That question — what structure survives a domain transition — is precisely what functors track. Category theory makes the tracking explicit and checkable.

The constraints implicit in the derivation — closure, opacity, and their companions — can be formalized as categorical constraints: conditions on what morphisms are permitted and how they compose. The question of whether such constraints are simultaneously satisfiable — whether a consistent model exists — is the kind of question category theory is built to answer.

The Geometry Maps to the Epistemology

S^2's topological features are not incidental to the epistemological framework — they generate it. No boundary: claims must track their validity conditions or drift into overreach, because there is no edge at which to stop. Uniform positive curvature: no position on the surface has intrinsic geometric privilege; the ground is isotropic, which is the geometric form of non-perspectival depth. $\pi_1 = 0$: consequence chains can close — the normative structure derived above. $\pi_2 = \mathbb{Z}$: the second homotopy group is non-trivial, meaning there exist non-contractible 2-spheres — integrative depth has structure above simple closure. This is why grounding is not merely more traversal of the same kind: it involves recognizing the topology of the whole surface, not just tracing paths on it.

What the Pairing Reveals

Trigonometry and category theory are not two perspectives on one problem. They are opposite limits. Trigonometry drives toward full precision — this surface, this curvature value, this invariant quantity, determined to computational limits with no ambiguity remaining. Category theory drives toward full generality — the entire space of possible structures and how they are mutually determined through relationships alone. Objects in a category are defined by their morphisms; there is no "intrinsic content" apart from relational structure. This level of abstraction is not optional: it is where you must stand to ask what holds across all possible domain transitions.

Neither limit is reachable from the other. Neither is complete alone. Together they recapitulate in mathematical form the duality the derivation began with: apprehended and apprehending, content fully determined and the structure of determination itself.

Computation and Verification

Universal computation was derived as the minimal carrier. It has a reflexive consequence: the derivation's own steps are machine-checkable.

Lean 4 is a proof assistant — software that verifies mathematical proofs by checking each logical step against a formal type theory. The human writes the proof; the machine verifies that every step follows from the preceding ones relative to explicit assumptions. If a step is invalid, the proof does not compile. The check is mechanical and deterministic.

This matters because of a general feature of knowing. Warrant is path-dependent: a claim's standing turns on the conditions under which it was reached, and evaluation without access to that path is structurally incomplete. This is not a limitation to be overcome — it is how knowing works. But formal verification is phase-invariant. A proof compiles or it doesn't. The state of consciousness of the prover is irrelevant to the check. The path by which the proof was discovered — whether through years of contemplative practice, a flash of insight, or systematic exploration — leaves no trace in the verification. This is not a minor technical convenience. It means the derivation has produced, from within itself, a mode of warrant that is phase-invariant. Verification checks proofs relative to a chosen formalization and explicit assumptions — not axiom adequacy, not semantic faithfulness. But within that scope, it is immune to the generating-state dependence that produces the characteristic closure-failures.

What formal verification establishes is proof-checking relative to explicit assumptions — not the adequacy of the formalization itself, which remains a genuine question requiring philosophical judgment. But within its scope, it eliminates the phase-dependence that generates those closure-failures.

The Equational Theories Project demonstrated what this enables at scale. The project mapped the implication graph among thousands of equational laws, resolving over 22 million implications within three months — contributions verified by machine, independent of contributor reputation (Tao et al., 2024–2025)[4]. What emerged was not just confirmed complexity but unexpected compressive structure: the complexity was apparent, the simplicity was actual. Simple equational laws generate enormous surveyable consequence spaces whose structure becomes visible only at scale. This is **emergent simplicity** — not proof of metaphysical emanation, but a concrete demonstration directly applicable to the derivation chain.

Each step of the derivation is the kind of claim that admits formalization. For any step that can be adequately formalized, verification eliminates the vulnerability: its validity does not depend on the generating state, the check is public, and the proof holds or it does not. The adequacy of formalization — whether formal structure faithfully captures structural insight — is constrained but not closed by formal tools.

Not all claims can be formalized. But the derivation chain that grounds the depth metric can, and that is sufficient.

That chain has been formalized and machine-checked in Lean 4. The artifact declares zero custom axioms and contains no `sorry`; every top-level theorem reduces to the three standard axioms underlying Mathlib itself. Its central steps are proved rather than assumed: $\pi_1(S^2) = 0$ via a Seifert–van Kampen development built from scratch, the dimension-1 exclusion ($\pi_1(S^1) \neq 0$) via covering-space path-lifting, and closure at the universal-computation threshold via Kleene's recursion theorem. The one irreducibly mathematical premise — the classification of surfaces, from which the uniqueness of S^2 follows — is carried as an explicit hypothesis the theorems quantify over, not smuggled in as an axiom (Close, 2026b).

The Derivation Chain

The preceding sections did not prepare for a derivation. They performed one. Recognition, mutual determination, reflexive stability, universal computation, geometric emergence, formal verification — each was derived from what preceded it, each preserved the structure of what it translated. What follows states the composition explicitly: the full chain from the primitive to embodied experience, with each step labeled by epistemic status.

Formally established:

> "there is" — apprehending and apprehended as pre-differentiated unity (*the primitive: undeniable, pre-subjective, already structured*) → recognition yields terms: meaning carries implicit perspectival frames, necessarily more than one (*structural analysis*) → mutual determination of

[4] See also Tao, T. (2026, February). *Mathematics in the age of AI* [Invited lecture]. https://youtu.be/mS9Lr43cIB4 — commentary on the project's implications for machine-assisted mathematics.

unity and multiplicity; particular terms have relative existence — what is actualized is rigorous epistemology, not the instantiations (*Plotinian analysis converging with pratītyasamutpāda*) → reflexive stability of mutual determination; generativity requires well-founded recursion; the primitive is a fixed point under self-application → ground condition (*entailment*) → work proceeds with invariant constraints across terms, not particular terms; the chain's own self-referential structure (fixed point, reflexive stability, well-founded recursion) requires Type 0 by Chomsky separation (*theorem*); ground identified with universal computation structurally (*convergence* + *minimality*) → Kleene's recursion theorem: closure is a theorem at the universal computation threshold; metric completion of discrete configuration space forced by convergence; recurrent orbits persist as compact invariant sets; consequence chains as loops, contractibility as closure without obstruction → $\pi_1 = 0$; dimension 1 unsatisfiable, dimension 2 first satisfiable, higher dimensions introduce unentailed structure; S^2 simply connected and dimension 1 excluded (*theorems*: $\pi_1(S^2) = 0$ and $\pi_1(S^1) \neq 0$, both machine-checked); that S^2 is the *unique* such surface follows from the classification of surfaces, carried as the derivation's single mathematical premise, not reproved here (Perelman confirms S^3 unique in dimension 3 — excluded by binding minimality)

Downstream availability:

The derivation does not terminate at S^2. The constraints implicit in the chain — that consequence chains can close, that no local position surveys the whole — narrow candidate physical dynamics in specific, formalizable ways. The details of that narrowing are beyond the scope of this paper. What matters here is that the chain from the primitive to S^2 is sufficient to ground the depth metric, and that the chain's continuation toward embodied physical experience is a live research program, not a promissory note abandoned at the point of difficulty.

The derivation finds these conditions — *finden*, not *erfinden*.

Two constraints bear directly on the failure modes. Closure: consequence chains *can* close — actions produce reactions that return to the agent — but not all do. Closure is possible, not guaranteed. Opacity: the full structure of the possibility space is not surveyable from any local position. There is no resolution at which the bounded encompasses the unbounded. Three characteristic closure-failures follow from these constraints. A failure of return: the consequence chain from a disclosure to action never closes back through ordinary functioning. A failure of tracking: the return-address from belief revision to its generating conditions is severed. A refusal of departure: the consequence chain from ordinary consciousness to non-ordinary disclosure is cut at the outset. In each case, the failure is not a feature of the ground but a local failure of consequence chains to close.

Ἀλήθεια

Heidegger recovered the word: ἀλήθεια, un-concealment, truth not as correspondence between proposition and fact but as the event in which what was structurally present but unavailable stands in the open (Heidegger, 1943/1998). Truth as verb, not noun. The activity of disclosure, singular.

The derivation performed this. Each step — from the primitive through mutual determination through computation through geometric emergence — was not a construction but an unconcealment: attending to what the terms already contained until the next structure presented itself. *Finden* is ἀλήθεια. The German carries the Greek without knowing it — finding as the act of encountering what was already there but unavailable to the prior frame.

Thoreau: “Morning is when I am awake and there is a dawn in me” (Thoreau, 1854). The dawn is not a time of day. It is the quality of attention in which unconcealment occurs. Walden is a two-year unconcealment experiment — strip away everything contingent, attend to what remains, live there, observe what it generates. His conclusion, that “in dealing with truth we are immortal,” is a precise report of what it is like to stand where the derivation lands: where structure holds by necessity rather than contingency, phase transitions do not threaten.

Emerson names the subjective signature: “To believe your own thought, to believe that what is true for you in your private heart is true for all men — that is genius” (Emerson, 1841). This is not narcissism. It is the recognition that when attention becomes transparent to structure — when private perception and universal form coincide — individual insight and formal necessity converge. The derivation provides the mechanism Emerson described but could not formalize.

The practitioner does not arrive at the sphere, or at simplicity, or at abstraction. The practitioner recognizes that embodied experience, in its full complexity, is consequence of the ground always already stood upon. The traditions preserved structural motifs that recur across independent transmission lineages — with the degree of genuine convergence itself an empirical question the framework is now positioned to investigate systematically. The derivation supplies the formal backbone of what they recognized.

Grounding the Depth Metric

The derivation chain is itself a canonical instance of integrative depth: each step includes and contextualizes all prior steps by formal necessity, not convention. Recognition includes the primitive. Mutual determination includes recognition. Reflexive stability includes mutual determination. The chain is a maximal totally ordered subset within the depth ordering — a spine that fixes the vertical scale. Perspectives branch at each node; distinct perspectives at the same depth may be incomparable, which is why depth is a partial order rather than a total one. The chain provides the vertical axis; perspectives spread horizontally at each level.

The integrative depth of a perspective is determined by the deepest node in the chain whose full structural content that perspective can represent without compression loss. Compression loss — the structural content lost when a claim is restated in another frame — becomes the operational measure of depth reduction: the distance between what a perspective claims to represent and what survives faithful translation. A perspective that can hold mutual determination but not reflexive stability has a specific, non-arbitrary depth. A perspective that can hold the full chain to S^2 has more — not by stipulation, but because the chain’s structure is formally derived.

This is the non-perspectival ground the opening required. The depth ordering is non-perspectival not by assertion but because the derivation chain is reached from every formal direction simultaneously — the Church-Turing convergence argument establishes that universal computation is not one formalism among many but the unique class at which all formalisms arrive. The depth metric inherits that non-arbitrariness. The postmodern objection — that any depth ordering covertly universalizes a particular perspective — fails against a chain whose derivation is overdetermined by independent formal considerations.

Structural Recovery of Theological Content

The derivation chain provides a non-partisan evaluation criterion for specific structural claims embedded in the world’s contemplative and theological traditions. The value is not vindication — that would be overreach wearing a cassock — but a common structural language that does not require accepting any tradition’s contingent premises. Some claims pass the test; some partially pass; some remain indeterminate. The variation is itself evidence that the framework discriminates rather than confirms.

A structural correspondence is non-accidental if it is predictable from the derivation's constraints without prior knowledge of the tradition. The test: given only the chain, would the traditional claim follow? Each mapping below opens with its verdict.

Plotinus (emanation): passes. The chain independently generates the structural logic of emanation; the correspondence is not fitted but found. Plotinus's sequence — the One generating Nous, Psyche, and Matter through successive overflow — maps to the derivation chain, and the mapping is not accidental (Plotinus, 1991). Plotinus had the argument form: the One is beyond being; it generates by necessity, not by choice; the generation proceeds through stages of increasing determination. In the derivation: "there is" maps to the One (bare existence beyond all attributes). The knowing co-given with existence, and the formal structures it generates, maps to Nous (Intellect). The successive stages of increasing determination map structurally; the specific physical content of those stages is part of the downstream program. A starting point beyond predication generating through stages of increasing determination is forced by the chain's structure. Plotinus had the structural logic. He did not have the mathematical content.

The Neoplatonic return — the soul's ascent back to the One — can be re-read as the recognition of the derivation chain from the other direction: starting from embodied experience and recognizing, through the full complexity, the generating unity. This is what the contemplative traditions describe as realization.

Nāgārjuna (two truths): passes. The derivation demonstrates form-emptiness identity structurally. The primitive — no attributes, no predication, surviving self-application precisely because there is nothing to deconstruct — is *śūnyatā* formalized. But this requires the reflexive move Nāgārjuna himself makes: *śūnyatā* is *śūnya*. The emptiness of emptiness. The primitive is not merely empty of attributes; it is empty of the kind of being that would give it *svabhāva* — own-nature, independent existence. And it survives self-application for exactly that reason: there is no fixed nature for the negation to land on. The fixed-point structure established above is not despite *śūnyatā* but because of it.

S^2 — determinate, fully characterized, arising entirely from the primitive's constraints — is *rūpa*. The chain from one to the other is not a departure from emptiness into form but the demonstration that they were never separate: the primitive generates the geometry; the geometry is what the primitive looks like when you attend to its structure. *Invariants* already established that all consequences have dependent origination — no term has independent existence; each arises in mutual dependence. S^2 is a consequence. It has exactly the *pratītyasamutpāda* status established above for all terms in the chain.

Conventional truth (*saṃvṛti-satya*) corresponds to the local view from any position — necessarily partial, necessarily perspectival, necessarily real within its generating conditions. Opacity captures this: no local view surveys the whole, yet the whole generates the local views.

The open question is whether positive formal characterization of a dependently originated structure installs *svabhāva* through the back door of precision. Precisely measuring a shadow does not give the shadow independent existence.

Augustine (privation): passes. From the chain alone, failure is an absence of closure rather than a positive feature — privation is forced by the structure, not fitted to it. Augustine's doctrine that evil is not a positive reality but an absence of good receives specific formal content through closure (Augustine, 397/1991). Structural failure is not a feature of the ground but a local failure of consequence chains to close. These failures are privations: not something added to the ground state but something missing from it — each an absence of closure, the return of a consequence chain to its generating conditions failing to complete.

Faith: passes — and this is the derivation's most original recovery. Faith operates in two senses that the derivation distinguishes precisely. The corrupted sense — belief held against evidence, obedience without understanding — is a gap-filler for missing steps. Where the chain has not been traversed, faith substitutes for structure. The recovered sense — what the traditions at their best pointed at — is the trust that the chain holds because one has traversed enough of it to recognize its structure. The difference is the difference between an unverified assertion and a theorem whose proof one has followed far enough to trust. The difference shows in the conditions. Gap-filler faith rests on validity conditions that have never been checked — the claim floats free of the conditions that would secure it. Structural trust rests on conditions partially traversed — the derivation has been followed far enough to recognize its structure, and the trust extends to what that structure entails beyond the point reached. The distinction is not between faith and reason. It is between faith that substitutes for structure and faith that recognizes it.

What the traditions called revelation — the sudden disclosure of structural truth — is the experience of traversing a derivation chain that actually closes. Not supernatural intervention. Not special access granted to some and denied to others. The chain standing in the open for anyone who traverses it. This is ἀλήθεια: truth as unconcealment, the event in which what was structurally present but previously unavailable becomes available.

Each contemplative and theological tradition is a partial traversal of the derivation chain, encoded in tradition-specific containers — vocabulary, practices, institutions, lineages. They could not prove the map was non-perspectival because they lacked the formal chain. The derivation provides a non-partisan basis for comparing traditions by the degree to which their structural claims preserve the chain's formal relations. That comparison has not yet been executed. But the framework for it now exists.

The structural insights that survived transmission across centuries — Plotinus on emanation, Nāgārjuna on two truths, Augustine on privation — are recoverable as specific formal features of the derivation chain. Not replaced. Not transcended. Grounded. Some structural motifs recurring across independent traditions were not arbitrary — they were working without the formal chain, and some of what they preserved turns out to be recoverable. The chain from the primitive to S^2 is complete; its continuation toward embodied physical dynamics is a live research program.

The mutual-exclusion framework was the error: the assumption that theological insights from different traditions are in zero-sum competition for the same truth. The derivation chain provides a non-perspectival ground on which insights from any tradition that achieved genuine structural recognition can be recovered, compared, and composed without requiring acceptance of that tradition's contingent premises. What millennia of rigorous theological thinking produced becomes available to thinkers who would never have accepted the premises those insights historically required. The derivation does not crown theology. It liberates theological content from the exclusivist container that made it inaccessible to anyone outside that container. What was structural persists. What was contingent falls away. And the structural content turns out to be enormous.

Connecting the Formal to the True

A natural objection remains: formal verification checks that proofs compile, but compilation is syntactic — what connects it to truth?

Cavell answered this with the title of his first major work: *Must We Mean What We Say?* (Cavell, 1969). The question is not about sincerity. It is a discovery: we are bound by the commitments inherent in our utterances whether we notice them or not. Language carries its meaning in its structure, not as an addition from outside. We do not first speak and then attach meaning. The speaking *is* the commitment.

This applies to formal language with full force. The Benacerraf problem (Benacerraf, 1973) assumes mathematical knowledge requires causal connection to abstract objects, then notes that abstract objects admit no causal relations. The derivation does not resolve this by argument but by demonstration: each step attended to structure already present in the terms, not imported from an abstract domain. The formal content was found rather than constructed — disclosed by sustained attention to the conditions structuring concrete experience. The question of how we connect to abstract objects does not arise when the structure was never elsewhere. The reader can check each step of this process; the derivation is its own evidence that the bridge exists, because it just crossed it.

Cavell saw the same structure from the other direction: "If philosophy is esoteric, that is not because a few men guard its knowledge, but because most men guard themselves against it." Benacerraf poses the philosopher's version of this defense — installing an unbridgeable gap between formal structure and the world, then treating the gap as a problem to be solved rather than a premise to be questioned. Its contemplative counterpart is the refusal to cross toward non-ordinary disclosure, dressed as rigor. Both assume that what is not immediately available must be elsewhere. The derivation answers both in the same way: by arriving. The formal chain does not overcome the defense by force of argument. It makes the defense visible by providing a chain that anyone can check, that no one's state of consciousness can invalidate, and that stands in the open for anyone willing to follow it. The traditions could not do this because they lacked the formal chain. The formalization does not replace the traditions' insight. It removes the excuse for guarding against it.

When the contingent is stripped away, what remains binds.

Self-Application

The derivation demands to be turned on itself. Its central claim — that S^2 is the minimal geometric carrier the constraints determine — was found in a particular state of sustained attention. Does that provenance infect the claim? It does not, and the reason is the whole point. The claim's validity conditions — that $\pi_1(S^2) = 0$ and that dimension 1 is excluded, both proved; that S^2 is the *unique* such surface, given the classification of surfaces; the minimality argument that fixes the dimension — are checkable from any phase. The proof compiles regardless of the prover's state of consciousness. Generation was bound to the state it occurred in; the conditions are not.

But here the derivation meets its own limit, and meets it honestly. That the conditions are phase-invariant is not the same as their being *adequate* — whether the formal structure faithfully captures the insight it claims to ground. That cannot be judged from inside the formalism; it requires holding the formal chain and the experiential content at once, and no verification settles it. The ground is checkable; the adequacy is not surveyable. This is the non-closure the derivation has been describing, now turned on the derivation itself: closure is available — the proof closes — and the whole is not surveyable from any position that could certify the fit.

That is the structural answer to the suspicion that any claim to a non-perspectival ground covertly smuggles a perspective back in. The suspicion assumes phase-invariance must be either total or illusory. The derivation shows a third thing: it is *locally specifiable*. The claim's conditions are phase-invariant; its adequacy is perspectival; and the boundary between them is a characterizable feature of the claim, not a vague concession. The critic is right that adequacy is perspectival — the derivation never said otherwise. What it establishes is that the conditions are not, and that a ground can be reached which is checkable without being surveyable. That is what it is for non-closure to hold at the root: the place you can stand and be certain is not a place from which you can see the whole.

Conclusion

The chain runs from *there is* to S^2 without a step that was chosen rather than forced. What was doubted away is the subject; what remains is a ground no perspective authored and every perspective presupposes. That ground is checkable — the proof compiles, and the state in which it was found leaves no trace in the check. It is not, for all that, surveyable: the adequacy of the formalism to what it formalizes is the one thing the derivation cannot close from inside, and the honesty of leaving it open is the derivation's own subject turned back on itself. Descartes was right to begin from what survives doubt. He began one step too late. Begin earlier, and the ground is found — *finden*, not *erfinden* — standing in the open for anyone who traverses it.

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