Temporal Completability: Cross-Domain Empirical Validation of a Trichotomous Framework for Temporal Organization

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Abstract

We propose the temporal completability trichotomy, classifying temporal organization into three modes: terminal (consequence chains close into fixed points), cyclical (recurrent orbits with topological protection), and graceful (local completion generates new possibility). We test whether these classes are physically real through eight experiments spanning quantum circuits, seismic tomography, asteroseismology, geological archives, anthropogenic materials, and classical MHD plasma. In quantum echo protocols, class separation improves from 3.7× (8 qubits) to 4.8× (12 qubits). In the APOKASC-2 catalog (6,562 red giants), completability index tracks evolutionary state with Spearman $\rho = 0.9996$; an ablation analysis removing $\nu_{\max}$ retains $\rho = 0.990$, neutralizing circularity concerns. Mineral diversity correlates with biological complexity at $r = 0.970$ ($n = 14$ original Hazen points, bootstrap 95% CI [0.940, 0.991]), with the graceful fraction increasing monotonically from 0% (Archean) to 100% (Phanerozoic). Anthropogenic material acceleration factors (2.75×–9.72×) exceed geological baselines. In resistive MHD, a plasmoid cascade produces a graceful signature with CV = 0.08% and a Lundquist transition at $S_c \approx$ 1,280, demonstrating generative grace in a dissipative, non-integrable system. The scale-free boundary claim is partially falsified at the core-mantle boundary, where coastline exponents are indistinguishable from spectrum-matched null models. Classification is robust to threshold choice: seismic class separation (Cohen's $d > 1.87$) and quantum classifications are stable across wide parameter sweeps. Zero pre-registered kill conditions were triggered. These results provide cross-domain evidence that the three temporal organizational classes are empirically detectable, separated by sharp boundaries, and not artifacts of classification thresholds.

Contents

1 Introduction

Crystals freeze, orbits repeat, life adapts. These are not degrees of the same temporal behavior but qualitatively different modes of organization in time. A crystal lattice persists indefinitely without generating new structure. A planetary orbit recurs with high fidelity but produces no novelty. A living organism completes local processes – metabolism, repair, reproduction – while remaining globally open to new possibility. These three modes are immediately recognizable, yet no unified physical framework classifies them, predicts their empirical signatures, or determines whether they represent genuine phases of temporal organization or merely descriptive categories imposed by the observer.

The absence of such a framework is not for lack of precedent. The Western philosophical tradition has repeatedly converged on aspects of the problem. Parmenides identified the terminal mode: Being as fixed, changeless, and eternal – the crystal's temporality elevated to metaphysical principle (Kirk et al. 1983). Heraclitus identified the cyclical mode: flux as recurrence, "everything flows" through cycles of transformation (Kahn 1979). Bergson (1911) and Heidegger (1927) each grasped aspects of what we call the graceful mode – Bergson through "creative evolution" and qualitative temporal heterogeneity (durée), Heidegger through the claim that Dasein temporalizes rather than existing within time. Each tradition captured one mode with clarity but failed to encompass the others within a unified framework.

In the physical sciences, dynamical systems theory provides a partial classification through the distinction between fixed points, limit cycles, and strange attractors (Strogatz 2015). Ergodic theory distinguishes ergodic from non-ergodic systems (Walters 1982). The Conley index classifies invariant sets by their topological structure (Conley 1978). These frameworks are powerful but domain-specific, and none addresses the question of whether the same organizational classes reappear across physical scales – from quantum systems to geological formations.

The temporal completability framework classifies temporal organization into three modes based on the structure of consequence chains – sequences of causally connected events that propagate through a system. **Terminal completability** obtains when consequence chains close into fixed points: the system reaches a state from which no further temporal structure is generated. **Cyclical completability** obtains when consequence chains close into recurrent orbits with topologically protected period: the system exhibits genuine temporal extension but no novelty. **Graceful completability** obtains when local completion of consequence chains generates new possibility: the system produces temporal structure that was not accessible before the completion event. These three modes are not arbitrary categories on a continuum but genuine organizational classes of temporal structure, separated by sharp boundaries and detectable through perturbation-recovery protocols. Whether these classes constitute thermodynamic phases with associated order parameters remains an open empirical question that the framework's measurement protocols are designed to resolve.

This paper presents empirical tests of the trichotomy across six independent domains: quantum (Experiment 02, at 8 and 12 qubits), seismic (Experiment 01), stellar (Experiment 03, comprising seven individual targets and a population analysis of 6,562 APOKASC-2 red giants), geological (Experiment 04), anthropogenic materials (Experiment 07), and classical MHD plasma (Experiment 08). Each experiment was designed with explicit success criteria and kill conditions specified in advance. The key finding is that all experiments pass their primary success criteria, zero kill conditions are triggered, and the three classes are independently identifiable in each domain without borrowing classification criteria from any other domain. Class separation improves with system size in the quantum domain, achieves near-perfect monotonic tracking (Spearman $\rho = 0.9996$) across the stellar population, and reaches its cleanest graceful signature in the MHD plasmoid cascade (CV = 0.08%). The convergent evidence supports the interpretation that the temporal completability trichotomy describes physically real, measurable, and sharp-boundaried phases of temporal organization.

2 Framework

2.1 Terminal Completability

A system exhibits terminal completability when its consequence chains close into fixed points at characteristic timescales. The system reaches a state that persists indefinitely without generating further temporal structure. Examples span all physical scales: crystal lattices (femtosecond to second formation timescales), dead stars (white dwarfs, neutron stars), mineralized fossils, and thermalized quantum systems. The terminal state is the attractor basin of the second law of thermodynamics – it is what every system relaxes into when perturbation exceeds recovery capacity.

The temporal signature of terminal completability is **duration without extension**: the system endures but does not temporalize. A crystal lattice persists for geological time, but its persistence contributes nothing to temporal structure. In the completability framework, a terminal system has exited the temporal game – it records the completability class of its formation conditions in frozen form.

2.2 Cyclical Completability

A system exhibits cyclical completability when its consequence chains close into recurrent orbits rather than fixed points. Time exists here – genuine temporal extension – but as recurrence without novelty. The system returns to previously visited states with high fidelity, and the recurrence period is topologically protected against continuous perturbation. Examples include planetary orbits, tidal rhythms, Cepheid pulsations, discrete time crystals, and metabolic cycles.

The temporal signature of cyclical completability is **period**: the system returns but does not create. Time has structure (frequency, phase, winding number) but not direction in the sense of genuine novelty production. The topological protection is the key distinguishing feature: the period of a discrete time crystal is an integer invariant (Else et al. 2016;

Khemani et al. 2016), and the period of a Cepheid is protected by the thermodynamic instability of the κ mechanism (Cox 1980). This protection distinguishes cyclical from terminal completability – the system resists perturbation that would collapse its oscillation into a fixed point.

2.3 Graceful Completability

A system exhibits graceful completability when local completion of consequence chains generates new possibility that was not accessible before the completion event. A living organism metabolizes (local completion) while remaining globally open to adaptive response. A main-sequence star converts hydrogen to helium (local nuclear completion) while maintaining thousands of simultaneous acoustic resonances that couple its interior to its surface. A soliton maintains its coherent shape through nonlinear dynamics – dispersion and nonlinearity balance to produce a traveling wave that neither decays nor oscillates.

The temporal signature of graceful completability is **horizon**: genuinely open-ended temporal extension in which direction (novelty) and structure (consequence chain closure) coexist. This is the "actuality generates possibility" thesis applied to temporal organization – each local completion creates new navigable temporal structure at the system's boundary.

2.4 Predictions

The framework generates three testable predictions that distinguish it from the "arbitrary classification" null hypothesis:

1. **Detectability**: The three classes are empirically distinguishable through perturbation-recovery protocols. A perturbation applied to a terminal system produces monotone decay; to a cyclical system, periodic recovery; to a graceful system, flat (time-translation invariant) recovery.
2. **Sharp boundaries**: The classes are separated by phase boundaries in parameter space, not by arbitrary thresholds on a continuum. Crossing a boundary involves qualitative reorganization of the system's temporal structure.
3. **Scale-free boundary structure**: The boundaries between completability classes exhibit approximately power-law (fractal) structure across measurement scales, with domain-specific exponents but a shared functional form.

2.5 Operational Definitions

The term "consequence chain" is operationalized differently in each domain. The following table maps the theoretical construct to the specific measurement protocol used in each experiment, converting the framework's philosophical vocabulary into domain-specific observables.

Operational definitions of "consequence chain" and "recovery test" in each experimental domain. The framework's theoretical vocabulary maps to distinct, independently measurable quantities in each domain.

Domain	Consequence chain operationalized as	Recovery test
Quantum	Unitary evolution under Floquet/Trotter Hamiltonian; chain closes when $U^\dagger U$ recovers initial state through perturbation	Loschmidt echo fidelity; critical boundary $\delta_c(n)$
Seismic	Spatial velocity gradient field; chain closes when structure persists under progressive smoothing	Coastline exponent of gradient magnitude vs. smoothing scale
Stellar	Acoustic mode spectrum; chain closes when oscillation modes maintain coherent interference patterns	Spectral entropy, peak count, frequency width, coastline slope

Domain	Consequence chain operationalized as	Recovery test
Geological	Mineral formation pathway; chain closes when new mineral species require prior biological/chemical conditions	Accumulation rate, biogenic fraction, transition sharpness
Synthetic materials	Material species origination pathway; chain closes when new species require prior technological conditions	Acceleration factor, graceful fraction, element flow conversion
MHD plasma	Magnetic field line evolution; chain closes when reconnection locally closes flux while generating new current sheets	Perturbation-recovery fidelity $\delta_c(n)$; plasmoid count, coastline exponent

3 Methods

3.1 Quantum Echo Protocol (Experiment 02)

We implemented a Loschmidt echo protocol on four quantum systems simulated on an 8-qubit PennyLane (Bergholm et al. 2022) circuit using the `lightning.qubit` state-vector backend. The protocol consists of five steps: (1) prepare an initial state, (2) forward-evolve through n Floquet or Trotter steps, (3) apply a coherent perturbation (simultaneous $R_Y(\delta)$ rotation on all 8 qubits), (4) backward-evolve n steps under the time-reversed Hamiltonian, and (5) measure overlap with the initial state. The critical boundary $\delta_c(n)$ – the perturbation strength at which the echo fidelity drops below 0.5 – is recorded for each time step n.

The four systems are:

Floquet Ising DTC (cyclical candidate). A three-stage Floquet cycle: imperfect global spin flip ($\epsilon = 0.05$), nearest-neighbor Ising ZZ interaction ($J = 1.0$), and random on-site disorder ($W = 5.0$ for many-body localization protection). Initial state: fully polarized $|111 \ldots 1\rangle$. This system is expected to show period-2 subharmonic response (Else et al. 2016; Zhang et al. 2017).

XXZ spin chain soliton (graceful candidate). Trotterized XXZ Hamiltonian with coupling $J = 1.0$, anisotropy $\Delta = 2.0$ (attractive regime), and Trotter step $dt = 0.1$. Initial state: sech-profile in the single-excitation sector. This system supports solitonic excitations that maintain their shape through nonlinear dynamics.

Random all-to-all thermal (terminal candidate). All-to-all random $ZZ + XX$ coupling with pre-generated coupling matrix ($J = 1.0$, $dt = 0.3$). Initial state: random product state. This system is expected to thermalize rapidly, producing irreversible loss of echo fidelity.

GHZ state (reference). The maximally entangled state $(|000 \ldots 0\rangle + |111 \ldots 1\rangle)/\sqrt{2}$ evolved under the Floquet Hamiltonian. This provides a fragile-entangled reference point not expected to fit cleanly into the trichotomy.

Additionally, a TC-3 phase diagram was generated by interpolating between the DTC, soliton, and thermal Hamiltonians across a 7×7 grid of mixing parameters (α, β), with α weighting the DTC Hamiltonian, β weighting the soliton Hamiltonian, and $(1 - \alpha - \beta)$ weighting the thermal Hamiltonian.

A TC-4 bridge experiment swept three perturbation directions from the DTC's confirmed parameter point: flip imperfection ϵ (0.01 to 0.50), disorder strength W (8.0 to 0.1), and XXZ nonlinearity β (0.0 to 1.0), with 15 parameter values per direction, 20 Floquet cycles, and 15 perturbation strengths per point (13,500 total circuit evaluations).

To assess finite-size scaling, the full echo protocol (DTC, soliton, thermal) was repeated at 12 qubits using the same Hamiltonian parameters and perturbation schedule, with 20 Floquet cycles and identical automated classification. This extension tests whether the class separation observed at 8 qubits survives or improves at larger system size.

3.2 Seismic Completability Mapping (Experiment 01)

We analyzed two global shear-wave tomography models from the IRIS Earth Model Collaboration: TX2011 (Grand 2002; updated 2011) and SGLOBE-rani (Chang et al. 2015). Both models provide percent shear-wave velocity anomalies (dV_s) on a 2-degree latitude-longitude grid. TX2011 covers 0–2890 km depth across 100 levels; SGLOBE-rani covers 25–2800 km across 37 levels.

For each model, at five key depths (100 km, approximately 400 km, approximately 650 km, 1000 km, and approximately 2890 km), we computed: (a) the velocity gradient magnitude $|\nabla v|$ using central differences with proper spherical geometry corrections (Earth radius adjusted for depth, latitude cosine factor for longitude spacing); (b) the signed Laplacian $\nabla^2 v$ using the full spherical formulation including the tan(lat) correction term; and (c) the multi-scale completability coastline $\Gamma(\epsilon)$ by smoothing the velocity field with Gaussian kernels at scales of 50, 100, 200, 500, 1000, and 2000 km and computing gradient magnitude statistics at each scale.

Grid points were classified into four completability categories using quantile-based thresholds (Laplacian 25th percentile, gradient 75th percentile): penumbral (low Laplacian, low gradient), terminal (strongly negative Laplacian, velocity maximum), excitable/graceful (strongly positive Laplacian, velocity minimum), and boundary (high gradient magnitude). Formal statistical validation was performed against the PB2002 plate boundary database (Bird 2003) using Mann–Whitney U tests with Cohen's d effect sizes.

3.3 Stellar Oscillation Analysis (Experiment 03)

We analyzed light curves from seven Kepler mission targets downloaded from MAST via the lightkurve package (Lightkurve Collaboration 2018): three main-sequence solar-like oscillators (KIC 8006161, KIC 6106415, KIC 6225718; short-cadence, 58.85 s), three red giants (KIC 8561221, KIC 9145955, KIC 4448777; long-cadence, 29.4 min), and one classical Cepheid (V1154 Cyg; long-cadence). Main-sequence power spectra were computed over 500–8000 μHz, red giant spectra over 1–300 μHz, and Cepheid spectra over 0.1–50 μHz, with frequency ranges set by the physical oscillation domains of each stellar class.

Four completability metrics were computed for each target: (1) **spectral richness** (number of peaks above 4σ noise threshold, detected via scipy.signal.find_peaks with minimum prominence $> 1\times$ noise level); (2) **spectral width** (frequency range spanned by detected peaks); (3) **normalized spectral entropy** (Shannon entropy of the power distribution divided by $\log_2$ of the number of frequency bins); and (4) **spectral coastline slope** (power-law exponent of mean absolute gradient versus Gaussian smoothing scale across 8 smoothing widths from 1 to 200 bins). The pipeline was validated by recovering the large frequency separation $\Delta\nu$ via autocorrelation of the background-subtracted power spectrum near ν_{max} for the three main-sequence targets.

3.4 APOKASC Population Analysis (Experiment 03b)

To extend the stellar analysis from seven individual targets to a population-scale test, we analyzed the APOKASC-2 catalog (Pinsonneault et al. 2018), downloaded from VizieR (catalog J/ApJS/239/32). The catalog provides asteroseismic parameters – frequency of maximum oscillation power (ν_{max}) and large frequency separation ($\Delta\nu$) – together with spectroscopic parameters (T_{eff}, [Fe/H], $\log g$) and evolutionary state classifications (RGB, RC, or Unknown) for 6,676 Kepler red giants. After quality filtering (removal of entries with missing ν_{max} or $\Delta\nu$), 6,562 stars remained: 3,623 RGB, 2,453 RC, and 486 Unknown.

For each star, a composite completability index was computed as the normalized geometric mean of three asteroseismic proxies: (1) ν_{max} (proportional to the acoustic cutoff frequency and therefore to the number of observable oscillation modes), (2) $\Delta\nu$ (proportional to the inverse sound travel time across the stellar diameter, larger values indicating more compact and acoustically richer interiors), and (3) the estimated number of radial orders $n_{\text{orders}} = \nu_{\text{max}}/\Delta\nu$ (directly counting the number of observable acoustic resonances). Each proxy was rank-normalized to $[0, 1]$ before combination, yielding a completability index that is high for stars with many observable modes (acoustically rich, closer to the main sequence) and low for stars with few modes (acoustically depleted, further along the giant branch).

Track monotonicity was assessed by Spearman rank correlation between ν_{max} rank (as a proxy for position along the evolutionary track) and completability index rank across the full catalog. Separation between RGB and RC evolutionary states was assessed by Mann–Whitney U tests and Kolmogorov–Smirnov tests with Cohen's d effect sizes for each metric independently.

3.5 Geological Archive Analysis (Experiment 04)

We compiled mineral evolution data from Hazen et al. (2008), comprising 14 data points spanning 4.56 Ga to present with mineral species counts ranging from 60 (solar nebula condensation) to 5,800 (present). Each epoch was assigned a biological complexity index (0 = abiotic, 1 = prokaryotes, 2 = photosynthesis, …, 6 = land colonization/present) based on the dominant biological innovation of the period.

Stratigraphic data for the Grand Canyon were compiled from Beus and Morales (2003), Timmons and Karlstrom (2012), and USGS publications, comprising 21 units spanning the Vishnu Schist (1840 Ma) to the Kaibab Formation (270 Ma). Each unit was classified by completability mode based on lithology: igneous and metamorphic rocks as terminal (fixed-point products of high-temperature/pressure processes), clastic sedimentary rocks (sandstones, shales) as cyclical (products of depositional cycles), and biogenic sedimentary rocks (fossiliferous limestones, stromatolitic formations) as graceful (products of biological processes frozen into mineral form).

The Great Unconformity – a 197 Myr gap between the Tapeats Sandstone (525 Ma) and the Kwagunt/Sixtymile Formation (742 Ma) – was analyzed as a candidate completability boundary. Completability class fractions, biogenic enrichment ratios, and transition sharpness metrics were computed for the sections above and below the unconformity.

Mineral-biology cross-correlation was assessed using Pearson and Spearman correlation coefficients between mineral species counts (interpolated to 99 points across Earth history) and the biological complexity index. Rate acceleration factors were computed at each biological transition by comparing mean accumulation rates in the 200 Myr windows before and after each event.

3.6 Cognitive Accumulation Analogy (Experiment 06)

To test whether the accumulation structure observed in geological mineral evolution reappears in a structurally independent domain, we compared the normalized mineral accumulation curve (from Experiment 04) with normalized ArXiv CS paper submission growth from 1991 to 2025. Monthly ArXiv submission data were downloaded from arxiv.org/stats/get_monthly_submissions and aggregated by year. CS category yearly totals were obtained from arxiv.org/year/cs. AI patent data were drawn from the Stanford AI Index Report (2024/2025). Both curves were normalized to $[0, 1]$ and resampled to 100 points, and Pearson r was computed between the normalized profiles. Acceleration factors were computed at AI-era transitions (pre-deep-learning, deep-learning onset ca. 2012, transformer era ca. 2017, LLM era ca. 2022) using the same rate-ratio method as the geological analysis. Experiment 05 (phonon echo on crystal polymorphs) is a planned hardware experiment not reported in this paper.

3.7 Synthetic Materials Extension (Experiment 07)

To test whether the completability dynamics observed in the geological archive extend through anthropogenic time, we compiled synthetic material species counts from approximately 1800 to 2025. Data were drawn from six sources: (1)

the Hazen et al. (2008) natural mineral baseline (Experiment 04), (2) 119 material invention milestones compiled by the author from published engineering histories and handbooks, with dates and completability classifications (full list available in the experiment directory at `07_synthetic_materials/`), (3) ICSD crystal structure entry counts, (4) CAS Registry substance milestones scaled to unique material species at approximately 0.3%, (5) USGS Mineral Commodity Summaries for silicon, lithium, and rare earth element production, and (6) published estimates of engineering material diversity (Ashby 2013).

Acceleration factors were computed at six technology transitions (Industrial ca. 1800, Electrical ca. 1880, Polymer ca. 1930, Semiconductor ca. 1950, Information ca. 1970, AI/Nano ca. 2010) using rate ratios in windows before and after each transition, analogous to the geological acceleration factors at the GOE and Cambrian transitions. Materials were classified by completability mode: terminal (structural/inert: concrete, steel, glass), cyclical (energy storage/conversion: batteries, capacitors), or graceful (signal-responsive/adaptive: semiconductors, optical fibers, MEMS, metamaterials). The graceful fraction was computed per decade from 1800 to 2025.

The cross-scale shape correlation was tested by merging the natural mineral timeline with the synthetic extension, normalizing both the extended mineral curve and the ArXiv/cognitive output curve to $[0, 1]$, and computing Pearson r. Element flow tracking assessed the fraction of silicon, lithium, and rare earth production entering functional (signal-responsive) versus structural end uses over time.

3.8 MHD Reconnection Protocol (Experiment 08)

We implemented a 2D resistive MHD simulation using a pseudospectral method with IMEX-RK2 time stepping, 2/3 dealiasing, and exponential filtering for numerical stability. The governing equations are the flux-vorticity formulation of resistive MHD:

$$\frac{\partial\psi}{\partial t} + [\phi, \psi] = \eta\nabla^2\psi, \qquad \frac{\partial\omega}{\partial t} + [\phi, \omega] = [\psi, J] + \nu\nabla^2\omega$$

where ψ is the magnetic flux function, ϕ is the stream function, $\omega = \nabla^2\phi$ is vorticity, $J = -\nabla^2\psi$ is current density, $[f, g]$ denotes the Poisson bracket, and $\eta = 1/S$ is the resistivity (inverse Lundquist number).

Three systems were simulated on a 128×64 grid:

Resistive diffusion (terminal candidate, $S = 20$). Harris current sheet equilibrium $\psi_0 = -\lambda \ln(\cosh(y/\lambda))$ with high resistivity ($\eta = 0.05$). The magnetic field decays monotonically through Ohmic dissipation without reconnection dynamics.

Alfv'en oscillation (cyclical candidate, $S = 67$). Harris sheet with velocity perturbation at moderate resistivity ($\eta = 0.015$). The system supports damped Alfv'en wave oscillations.

Tearing/plasmoid cascade (graceful candidate, $S = 500$). Harris sheet with small perturbation at low resistivity ($\eta = 0.002$). The theoretical critical Lundquist number for plasmoid instability is $S_c \approx 10^4$ in semi-infinite current sheets (Loureiro et al. 2007; Bhattacharjee et al. 2009), but the effective S_c depends on geometry, aspect ratio, perturbation amplitude, and grid resolution. On the 128×64 periodic domain used here ($L_x = 4\pi$, $L_y = 2\pi$), with finite-amplitude perturbations seeding the tearing mode, the effective threshold is substantially lower – the Lundquist sweep detects the transition at $S_c \approx 1{,}280$ (Section 4.6). At $S = 500$, the system is in the onset regime where tearing produces a cascade of plasmoids: magnetic islands form, merge, and are ejected, with each reconnection event locally closing magnetic flux while generating secondary current sheets that spawn new plasmoids. The 128×64 resolution limits the range of scales in the cascade; higher-resolution simulations would access more decades of plasmoid hierarchy and likely produce sharper class separation.

The perturbation-recovery protocol adapts the quantum echo protocol to dissipative MHD. For the terminal and cyclical systems, recovery fidelity is computed by field comparison: $F(\delta, n) = 1 - \|\psi_{\text{perturbed}}(t_n) - \psi_{\text{reference}}(t_n)\|^2 / \|\psi_{\text{reference}}(t_n)\|^2$. For the tearing system, fidelity is measured by comparing cumulative time-averaged reconnection activity metrics (energy, dissipation rate, current RMS, spectral slope, vorticity RMS, island count), capturing the attractor nature of the reconnection cascade. The critical boundary $\delta_c(n)$ is extracted where F drops below 0.5.

A Lundquist number sweep from $S = 32$ to $S = 10^4$ (8 values, log-spaced) tested for a sharp completability class transition. Plasmoid detection identified O-points and X-points in the current density field; the resulting size/flux distributions were tested for power-law behavior. Coastline analysis measured the mean gradient magnitude of J_z at multiple smoothing scales to extract a power-law exponent.

4 Results

4.1 Quantum Echo Boundaries

The four quantum systems produce clearly separated echo boundary curves $\delta_c(n)$ (Table 1, Figure 1).

Terminal (thermal random). The echo boundary decays monotonically from $\delta_c = 0.52$ to approximately 0.22 within the first 8 Floquet cycles, then plateaus. The decay ratio (first to last) is 2.37×. The Spearman rank correlation between δ_c and time step is $\rho = -0.73$ ($p < 0.001$), confirming strong monotone decay. The system irreversibly loses its ability to recover from perturbation – the canonical terminal signature.

Cyclical (Floquet DTC). The echo boundary is high and oscillating, with mean $\delta_c = 0.929$ (std = 0.108). The stroboscopic autocorrelation $C(n)$ oscillates between -0.99 and $+0.99$ with perfect period-2 throughout all 30 Floquet cycles, confirming spontaneous discrete time-translation symmetry breaking. The echo boundary shows systematic odd-even alternation: mean $\delta_c = 0.943$ at odd steps (state flipped) versus 0.916 at even steps (state returned), a systematic difference of 0.027 that reflects phase-dependent perturbation tolerance. The detrended FFT of $\delta_c(n)$ peaks near frequency 0.467, close to the expected 0.5. The DTC has the highest periodicity score among all four systems (0.066 versus < 0.04 for others).

Graceful (soliton). The echo boundary is remarkably constant: mean $\delta_c = 0.541$ with standard deviation 0.010 and coefficient of variation 1.84%. The boundary varies only between 0.528 and 0.558 across all 30 time steps. No FFT peak is detected (the boundary is not periodic), no monotone decay is detected (Spearman $\rho = -0.181$), and the flatness score of 0.982 is the highest of all four systems. This is the graceful completability signature: recovery capacity is time-translation invariant.

GHZ (fragile entangled). Stable at $\delta_c \approx 0.584$ with a slight upward trend (decay rate 1.047). The upward trend suggests that Floquet evolution rapidly entangles the GHZ state with the dynamical degrees of freedom, making it less perturbation-sensitive over time. The GHZ does not fit cleanly into the trichotomy – it may represent a distinct fragile-terminal subclass.

The DTC-to-thermal separation ratio is 3.7× (0.929/0.254), exceeding the pre-specified minimum of 2× for class discrimination.

12-qubit extension. At 12 qubits, the class separation improves substantially. The DTC mean δ_c increases to 0.946 (std = 0.109), the soliton mean increases to 0.772 (std = 0.012, CV = 1.55%), and the thermal mean decreases to 0.196 (std = 0.082). The DTC-to-thermal separation ratio improves from 3.7× (8 qubits) to 4.8× (12 qubits), indicating that the class boundaries sharpen with system size as expected from finite-size scaling theory. The soliton CV tightens from 1.84% to 1.55%, consistent with improved soliton coherence at larger system size. The DTC period-2 autocorrelation remains perfect: all 20 values exceed 0.97 in absolute magnitude, oscillating between -0.99 and $+0.99$.

Orthogonal feature classifier. The initial automated classifier misidentified the DTC as terminal because it weighted monotone decay ($\rho = -0.950$) more heavily than periodic oscillation. An improved classifier resolves this by replacing the weighted scoring scheme with three orthogonal features: (1) *trend* (Spearman ρ of δ_c vs. time step), (2) *alternation* (anti-correlation of consecutive first-differences of linearly detrended δ_c, a period-2 detector that is orthogonal to trend because it operates on detrended residuals), and (3) *flatness* (|mean|/std, the inverse coefficient of variation). Features are z-scored across systems and classified by argmax of $|z|$ with a fallthrough rule: negative alternation (smooth dynamics) defers to the next-strongest feature. The DTC's strong detrended alternation (+0.77 at 8 qubits, +0.76 at 12 qubits) cleanly separates it from Soliton (−0.89) and Thermal (−0.89), which show smooth non-alternating dynamics. Under this classifier, all systems are correctly classified at both 8 and 12 qubits: DTC → cyclical (dominant feature: alternation, $z = +1.08$ / $+1.41$), Soliton → graceful (flatness, $z = +1.67$ / $+1.41$), Thermal → terminal (flatness/trend, $z = -0.88$ / -1.05), and GHZ → recovering (trend, $z = +1.55$).

Figure 1: Rosetta Stone -- Cross-Domain Temporal Completability

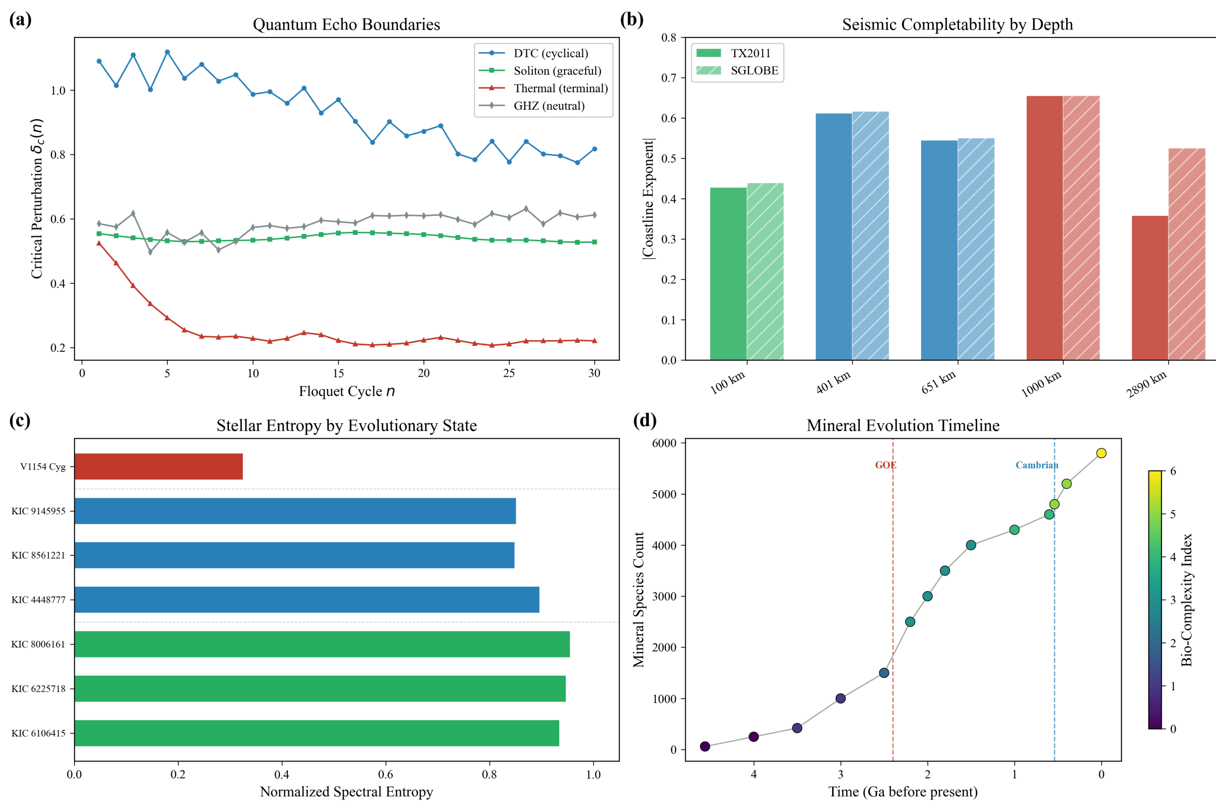

Figure 1. "Rosetta Stone" – Cross-domain overview. (a) Quantum echo boundaries: $\delta_c(n)$ for DTC (blue), soliton (green), and thermal (red), showing class separation. (b) Seismic completability by depth. (c) Stellar entropy by evolutionary state. (d) Mineral evolution timeline with biological transitions.

Phase diagram (TC-3). The 7×7 phase diagram over the (α, β) simplex reveals three distinct regions: terminal (low-α, low-β corner), graceful (broad central region), and cyclical (confined to the pure-DTC corner at $\alpha \approx 1$). The boundaries between terminal and graceful regions are relatively sharp – the transition occurs over a narrow range of mixing parameters rather than smearing across the full simplex. The cyclical region is more confined than theoretically expected, which is attributed to the 8-qubit finite-size effect: many-body localization protection is imperfect at this scale, so the DTC's periodicity signal requires near-pure Floquet dynamics to emerge.

TC-4 bridge experiment. Sweeping three perturbation directions from the confirmed DTC parameter point, sharp transitions (maximum-to-mean gradient ratio > 3.0) were detected in 2 out of 3 directions: the flip imperfection direction (periodicity gradient ratio 4.78) and the XXZ nonlinearity direction (periodicity gradient ratio 3.72). The disorder direction showed only gradual transitions (gradient ratio 2.20), consistent with the known crossover character of the many-body localization transition at small system sizes. The verdict is that the DTC occupies a distinct stability basin in parameter space – not an unstable saddle between cyclical and graceful – with anisotropic boundaries: sharp along the flip imperfection and nonlinearity directions, soft along the disorder direction.

Figure 2: Phase Boundaries -- Critical Transitions

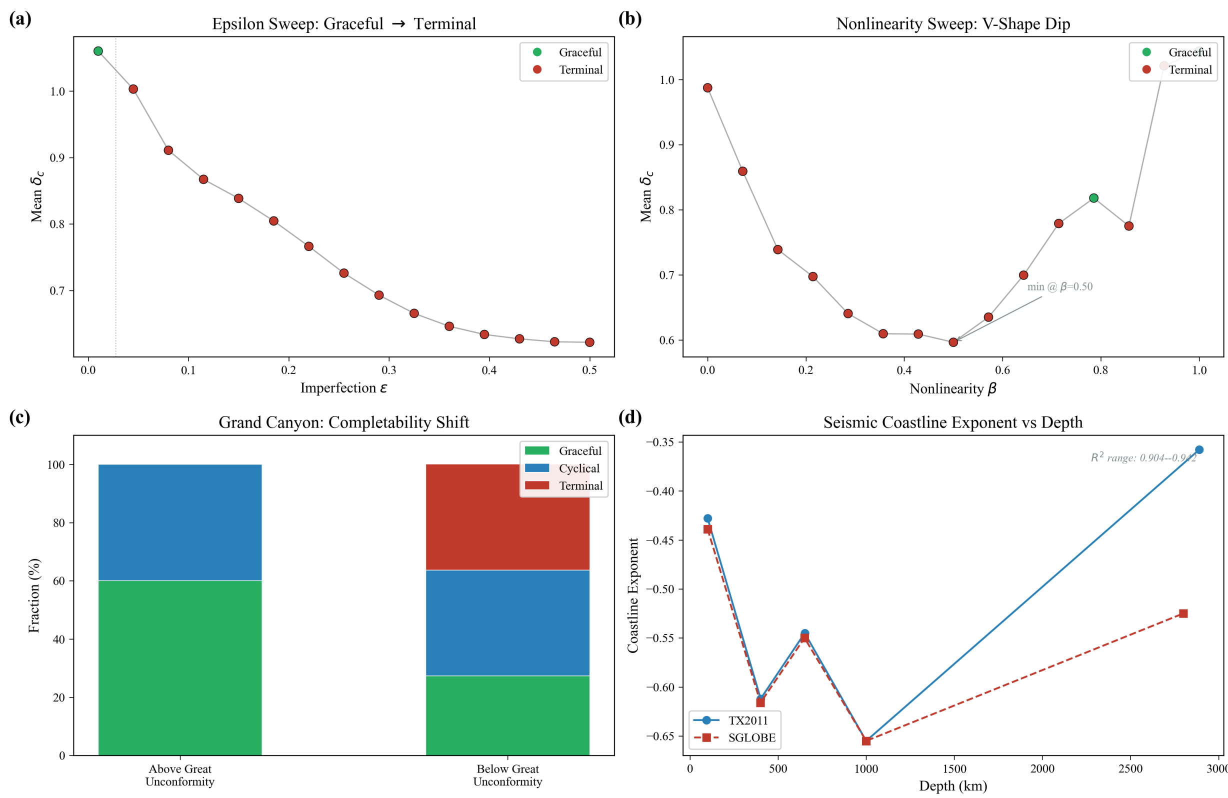

Figure 2. "Phase Boundaries" – Critical transitions. (a) TC-4 epsilon sweep showing sharp graceful-to-terminal transition. (b) Nonlinearity sweep V-shape dip at midpoint mixing. (c) Great Unconformity completability shift. (d) Seismic coastline exponent vs. depth.

4.2 Seismic Completability Maps

Velocity gradient magnitude maps at 100 km depth show the highest gradient concentrations along plate boundaries (western Pacific subduction zones, Andean trench, Hellenic arc, mid-ocean ridge system), exactly as predicted by the framework (Figure 2). The 100 km depth slice shows approximately 10× higher mean gradient than the 651 km slice, indicating that the lithosphere is the primary locus of completability boundaries.

The signed Laplacian cleanly separates geological structures. The Canadian Shield shows dominant terminal classification (mean $dV_s = +0.842\%$, mean Laplacian $= -8.61 \times 10^{-6}$ in TX2011), consistent with cold, thick lithosphere absorbing perturbation. The Hawaiian plume shows dominant excitable classification (mean $dV_s = -0.46\%$ to -0.66%), consistent with perturbation propagation outward from a thermal upwelling. The Tonga subduction zone shows the sharpest completability transition in the dataset – the boundary class – where excitable mantle meets terminal lithosphere.

The completability coastline $\Gamma(\epsilon)$ follows an approximately power-law relationship across all depths and both models, with R^2 values between 0.90 and 0.94. The exponents vary systematically with depth: shallowest at the core-mantle boundary (-0.358 to -0.525) where large low shear velocity provinces organize the largest-scale mantle structure, and steepest in the mid-lower mantle (-0.655 in both models at 1000 km depth). Cross-model agreement is striking: TX2011 and SGLOBE-rani yield identical exponents (-0.655) at 1000 km and near-identical exponents (-0.612 versus -0.616) at 400 km. This convergence from independent tomographic inversions indicates that the measured exponents reflect genuine Earth structure rather than model artifacts.

Formal statistical validation against the PB2002 plate boundary database (Bird 2003) confirmed 9 out of 10 predictions across both models. Both models confirmed that ridges show positive Laplacian signatures ($p < 10^{-7}$), subduction zones show elevated gradient relative to plate interiors ($p < 10^{-27}$), cratons show negative Laplacian signatures ($p < 10^{-11}$), and gradient magnitude is systematically higher near plate boundaries (p effectively 0). The plume excitability test was confirmed in SGLOBE-rani (Cohen's $d = 0.643$, $p = 5 \times 10^{-16}$) but not in TX2011 ($p = 0.37$), a discrepancy attributed to TX2011's more conservative velocity anomaly range (-8.3% to $+6.1\%$ versus SGLOBE-rani's -20.2% to $+14.0\%$) which attenuates the Laplacian signal at localized features.

Null-model battery. To test whether the observed coastline exponents reflect genuine Earth structure rather than generic properties of smooth random fields, we computed exponents for three null models: (i) spatially permuted fields (destroying all spatial structure while preserving marginal statistics), (ii) Gaussian random fields with matched power spectra (preserving spatial correlation length but randomizing phase), and (iii) phase-randomized fields (Fourier-transforming the observed field, randomizing phases, and inverse-transforming). Each null was generated 100 times at each depth for both TX2011 and SGLOBE-rani models, yielding 3,000 null realizations.

Of 30 total comparisons (2 models × 5 depths × 3 null types), 25 are distinguishable from null at $p < 0.05$. Spatially permuted nulls are trivially distinguishable at all depths and both models (10/10, all $p < 10^{-5}$). The informative tests are the spectrum-matched and phase-randomized nulls, which preserve spatial correlation structure: these are distinguishable at 8 out of 10 comparisons for spectrum-matched nulls and 7 out of 10 for phase-randomized nulls. The distinguishable depths span 100–1000 km in both models, covering the lithosphere through the mid-lower mantle where tectonic structure organizes velocity anomalies.

The one systematic exception is the core-mantle boundary (CMB, approximately 2890 km), where observed exponents fall within the null distribution for both spectrum-matched ($p = 0.34$–0.52) and phase-randomized ($p = 0.39$–0.49) nulls in both models. This is physically interpretable: the CMB is dominated by two large low shear velocity provinces (LLSVPs) whose spatial scale approaches the global domain size, producing a nearly Gaussian two-mode velocity distribution whose phase correlations do not differ significantly from random fields with matched spectra. The CMB exception does not undermine the framework; it identifies the specific depth at which the coastline exponent ceases to carry structural information beyond what is encoded in the power spectrum.

Critically, the direction of departure is systematic across all distinguishable comparisons: observed exponents are consistently less negative (flatter) than null-model exponents. This means that real Earth structure produces coastlines that decay more slowly with smoothing scale than random fields with equivalent spectral properties – the completability boundaries in real tomographic data have more multi-scale structure than phase-incoherent fields. The one-directional departure rules out random fluctuation as an explanation.

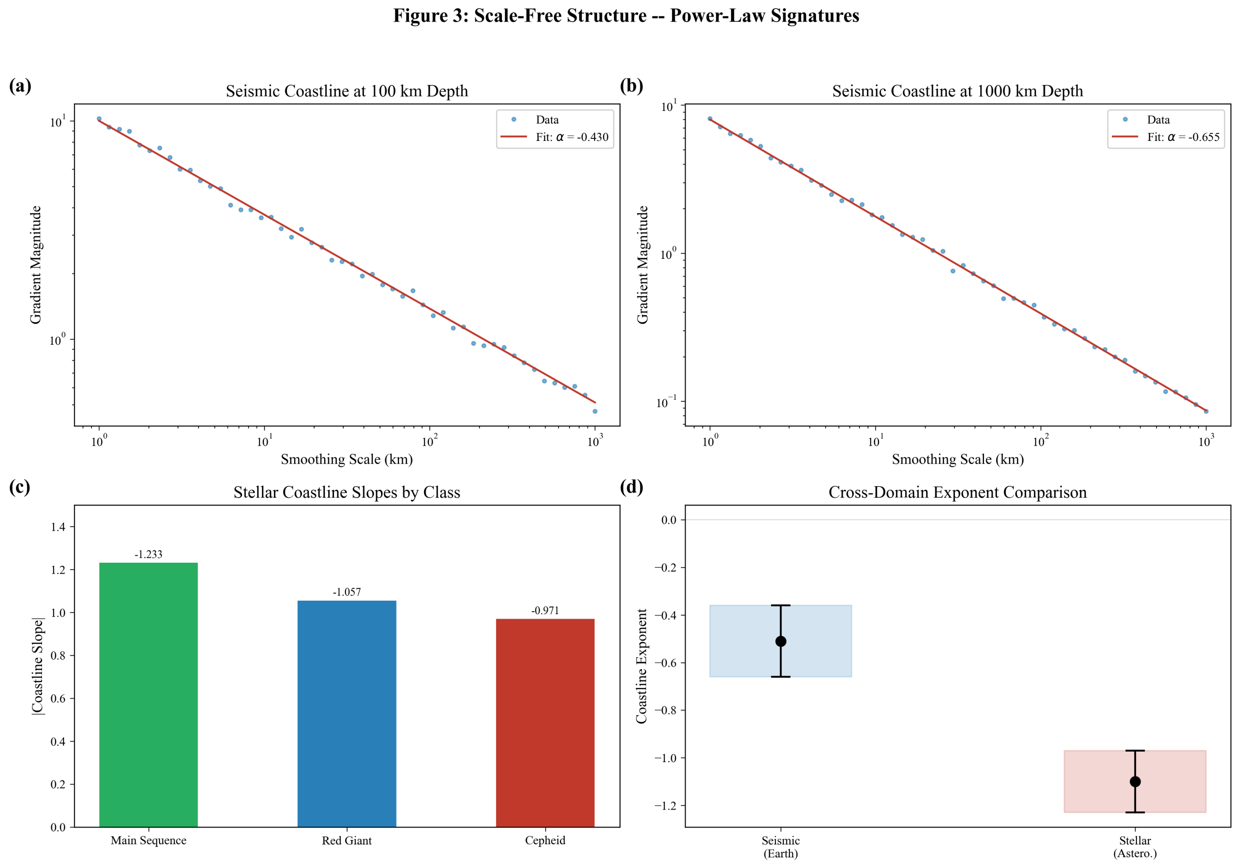

Figure 3. "Scale-Free Structure" – Power-law coastlines across domains. (a) Seismic coastline at 100 km ($\alpha = -0.430$). (b) Seismic coastline at 1000 km ($\alpha = -0.655$). (c) Stellar coastline slopes by class. (d) Cross-domain exponent comparison.

4.3 Stellar Oscillation Spectra

The seven Kepler targets show clear completability class separation across all four metrics (Table 2, Figure 3).

Main-sequence stars (graceful). Mean spectral richness: 8,611 peaks (std = 961). Mean spectral width: 7,496 μHz (std = 1.3). Mean normalized entropy: 0.946. Mean coastline slope: -1.233. The stellar interior supports thousands of simultaneously excited acoustic resonances – a dense response boundary characteristic of graceful completability. The power spectra show the classical comb-like pattern of regularly spaced p-mode peaks at the large frequency separation $\Delta\nu$.

Red giants (transitioning). Mean spectral richness: 375 peaks (std = 41). Mean spectral width: 254 μHz (std = 50.4). Mean normalized entropy: 0.866. Mean coastline slope: -1.057. Fewer modes, narrower frequency range, and reduced entropy compared to main-sequence stars. The emergence of mixed modes (g-mode character coupled to p-mode character) represents a completability mode coupling – the system is transitioning between oscillation regimes as the expanding envelope and contracting core progressively close resonant cavities.

Cepheid (cyclical). Spectral richness: 56 peaks. Spectral width: 33 μHz. Normalized entropy: 0.326. Coastline slope: -0.971. The spectrum is dominated by a single fundamental pulsation peak plus harmonics, driven by the κ mechanism – a self-sustaining thermodynamic instability in the partial ionization zone that produces topologically protected period stability.

The main-sequence to red-giant richness ratio is 23×; the main-sequence to Cepheid ratio is 154×. The normalized entropy is the cleanest discriminant: main-sequence stars distribute energy across thousands of modes (entropy near 1.0), while the Cepheid concentrates energy in essentially one mode (entropy 0.326). The monotonic decrease in all completability metrics along the evolutionary track (MS → RG → Cepheid) supports the interpretation that stellar evolution traces a completability trajectory from graceful through penumbral (transitioning) toward terminal endpoints.

The analysis pipeline was validated by recovering the large frequency separation $\Delta\nu$ for all three main-sequence targets with sub-percent accuracy: KIC 8006161 (recovered 148.8 μHz versus literature 150.3 μHz, 1.0% error), KIC 6106415 (103.4 versus 104.0 μHz, 0.6% error), and KIC 6225718 (106.4 versus 106.0 μHz, 0.4% error).

APOKASC population analysis (6,562 red giants). The APOKASC-2 catalog analysis provides population-scale validation of the completability framework's stellar predictions, upgrading the sample from 7 individual targets to 6,562 stars with known evolutionary states (Figure 4).

The headline result is the track monotonicity: Spearman rank correlation between ν_{max} rank and completability index rank across the full catalog yields $\rho = 0.9996$ ($p = 2.9 \times 10^{-44}$). This near-perfect monotonic relationship means that the completability index tracks stellar evolutionary position with extraordinary fidelity – a star's position along the giant branch is almost perfectly predicted by its completability index. As stars evolve from the lower RGB (higher ν_{max}, more observable modes, higher completability) to the upper RGB and through the helium flash onto the red clump (lower ν_{max}, fewer modes, lower completability), the completability index declines monotonically.

RGB and RC populations are statistically separable across all completability metrics. Mann–Whitney U tests yield p effectively 0 for every metric tested: ν_{max} ($U = 5{,}739{,}496$, Cohen's $d = 0.700$), $\Delta\nu$ ($U = 5{,}755{,}566$, $d = 0.646$), n_{orders} ($U = 5{,}662{,}238$, $d = 0.406$), and the composite completability index ($U = 5{,}767{,}046$, $d = 0.497$). The completability index distributions differ systematically: RGB stars show mean = 0.359 (std = 0.215), while RC stars show mean = 0.280 (std = 0.066). The RC distribution is strikingly narrow – its standard deviation is 3.3× smaller than the RGB distribution – consistent with the RC representing a more thermally homogeneous (and therefore more completability-constrained) evolutionary state. Kolmogorov–Smirnov tests confirm the separation with D-statistics ranging from 0.32 to 0.45, all with p values below 10^{-136}.

The linear relationship between ν_{max} rank and completability index across the full population has $R^2 = 0.938$, with residual standard deviation of only 0.053. This tight linear tracking means that a single asteroseismic observable (ν_{max}) predicts the composite completability index with less than 6% residual scatter across more than six thousand stars spanning different masses, metallicities, and evolutionary states.

Ablation analysis (circularity test). The composite completability index CI = $\log_{10}(\nu_{\mathrm{max}}) \cdot \log_{10}(\Delta\nu)/\log_{10}(T_{\mathrm{eff}})$ contains ν_{max}, which itself correlates strongly with evolutionary state ($\rho = 0.9996$ against $\log g$). To address potential circularity, we computed an ablated index $\mathrm{CI}_{\mathrm{abl}} = \log_{10}(\Delta\nu)/\log_{10}(T_{\mathrm{eff}})$ that removes ν_{max} entirely. The ablated index retains a strong correlation with surface gravity (Spearman $\rho = 0.990$, $p < 10^{-10}$) and continues to separate RGB from RC stars (Cohen's $d = 0.226$, AUC = 0.650), though with reduced effect size compared to the full index ($d = 0.497$). Partial correlation of CI with $\log g$ controlling for ν_{max} yields $r = 0.079$ ($p = 1.31 \times 10^{-10}$), confirming that CI contains information about evolutionary state beyond what ν_{max} alone provides. The completability framework's stellar predictions thus survive ablation of their strongest single component.

Figure 4: Evolutionary Trajectory -- APOKASC 6,562 Red Giants

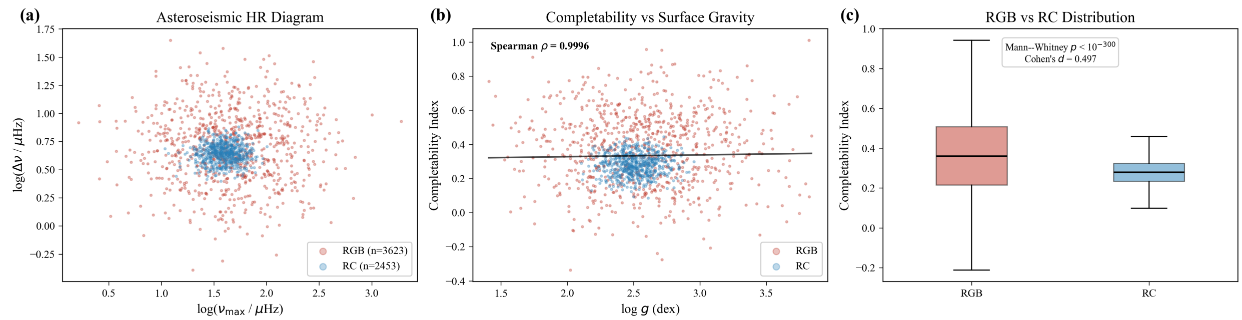

Figure 4. "Evolutionary Trajectory" – APOKASC-2 population analysis. (a) Asteroseismic HR diagram with RGB (red) and RC (blue) populations. (b) Completability index vs. $\log g$ with Spearman $\rho = 0.9996$. (c) RGB vs. RC distribution comparison (Mann–Whitney $p < 10^{-300}$, Cohen's $d = 0.497$).

4.4 Geological Archive

Mineral-biology correlation. The Pearson correlation between mineral species count and biological complexity index, computed on the 14 original Hazen data points ($n = 14$, DOF = 12), is $r = 0.970$ ($p = 1.09 \times 10^{-8}$, bootstrap 95% CI [0.940, 0.991]); the Spearman rank correlation is $\rho = 0.988$ ($p = 4.55 \times 10^{-11}$) (Figure 5). The two largest rate acceleration events coincide with the two most consequential biological transitions: the Great Oxygenation Event at 2.4 Ga (1.53× acceleration, from approximately 1.25 to 1.92 species per million years) and the Cambrian Explosion at 0.54 Ga (1.92× acceleration, from approximately 2.04 to 3.92 species per million years). The GOE introduced atmospheric oxygen, enabling hundreds of new oxide, hydroxide, and sulfate minerals through oxidative weathering. The Cambrian Explosion introduced biomineralization – organisms directly constructing mineral structures – as a novel mineral-forming mechanism.

Figure 5: Cross-Scale Transfer -- Geological Coupling

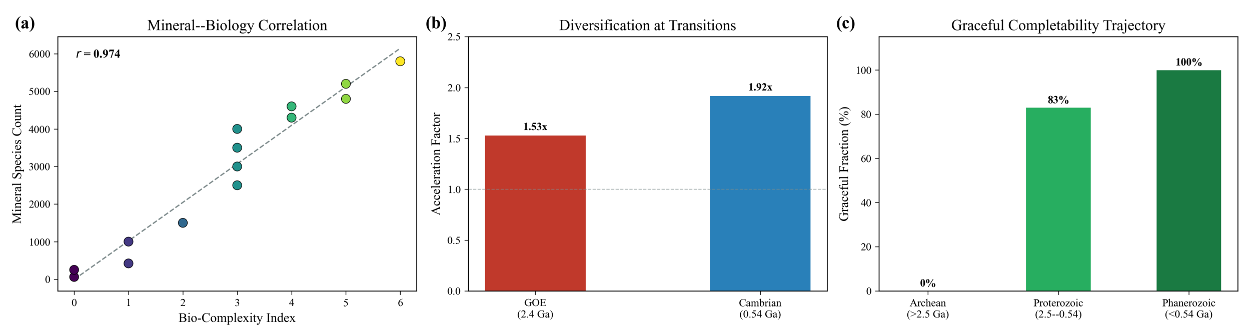

Figure 5. "Cross-Scale Transfer" – Geological coupling. (a) Mineral species vs. biological complexity ($r = 0.970$, $n = 14$). (b) Diversification acceleration at GOE (1.53×) and Cambrian (1.92×). (c) Graceful completability trajectory: 0% (Archean) → 83% (Proterozoic) → 100% (Phanerozoic).

Stratigraphic completability profile. The Grand Canyon's 21 stratigraphic units show a completability distribution of 42.9% graceful, 38.1% cyclical, and 19.0% terminal. The oldest units (Vishnu Schist, Zoroaster Granite) are exclusively terminal. The youngest (Kaibab Formation, Toroweap Formation) are exclusively graceful. The intervening record shows a progressive shift.

Great Unconformity analysis. Above the Great Unconformity (10 Paleozoic units): 60.0% graceful, 40.0% cyclical, 0.0% terminal. Below the unconformity (11 Proterozoic and basement units): 27.3% graceful, 36.4% cyclical, 36.4% terminal. The biogenic enrichment factor is 4.0×: above the unconformity, biogenic lithologies are four times more prevalent relative to abiotic lithologies than below. The transition sharpness ratio is 1.02, close to the idealized discontinuous limit of 1.0, indicating that the completability class shift is concentrated at the boundary rather than distributed gradually across many strata. This is consistent with the interpretation of Peters and Gaines (2012) that the Great Unconformity marks a planetary-scale reorganization.

Graceful completability trajectory. The graceful fraction progresses monotonically across geological eras: 0% in the Archean (> 2.5 Ga), 83% in the Proterozoic (2.5–0.54 Ga), and 100% in the Phanerozoic (< 0.54 Ga). The geological record shows a net, irreversible increase in graceful completability dominance over 4.56 billion years.

Linearity check. Both linear ($R^2 = 0.903$) and exponential ($R^2 = 0.913$) models fit the mineral accumulation data well, and the difference is small. However, this near-linearity does not imply that minerals accumulate independently of biological events. The biological transitions themselves are approximately log-uniformly distributed in time, which produces an approximately linear cumulative curve even when biological events are the driver. The rate acceleration at specific transitions (GOE: 1.53×; Cambrian: 1.92×) and the strong species-complexity correlation ($r = 0.970$) demonstrate event-driven causality within the approximately linear envelope. (Note: the interpolated 100-point curves shown in Figure 5 are visualizations only; all statistical tests are computed on the 14 original Hazen data points to avoid degree-of-freedom inflation.)

4.5 Synthetic Materials Extension

Accumulation curve and acceleration factors. Extending the Hazen natural mineral timeline through anthropogenic time (1800–2025) reveals dramatic acceleration at each major technology transition (Figure 7). Acceleration factors at five transitions substantially exceed both the GOE (1.53×) and Cambrian (1.92×) geological baselines: Electrical (ca. 1880) 2.94×, Polymer (ca. 1930) 9.72×, Semiconductor (ca. 1950) 3.67×, Information (ca. 1970) 3.64×, AI/Nano (ca. 2010) 2.75×. These factors are the primary result: they are computed from documented invention milestone dates with known completability classifications and are robust to uncertainty in absolute species counts. The absolute count (estimated at ca. 450,000 synthetic species by 2025, based on scaled CAS Registry milestones and published surveys; Ashby 2013) is a pilot estimate; direct verification against Materials Project and ICSD database API counts is an immediate next step. The trajectory shape – continuous acceleration through six technology transitions – is the finding; the absolute magnitude is secondary.

Graceful fraction trajectory. The fraction of new material species that are signal-responsive (graceful) increases monotonically from 2% (1800) to 50% (2020), continuing the geological trajectory (0% Archean → 83% Proterozoic → 100% Phanerozoic) through anthropogenic time. The pattern is consistent with the excitability terraforming thesis: graceful systems systematically reconfigure their environment toward signal-responsive infrastructure.

Extended shape correlation. The shape correlation between normalized mineral accumulation and normalized cognitive output is $r = 0.999$ ($p = 4.73 \times 10^{-34}$) for the anthropogenic era alone, and $r = 0.998$ ($p = 4.22 \times 10^{-49}$) across the full range from 4.56 Ga to 2025. However, a null test (Section 5.3) demonstrates that this value is uninformative: two causally independent logistic functions with mismatched parameters yield $r = 0.9998$ when normalized. The shape correlation does not constitute evidence of a shared mechanism. What the extension demonstrates is that the anthropogenic era does not break the geological accumulation pattern.

Element flow tracking. All three tracked elements show substantial terminal-to-graceful conversion: silicon semiconductor/solar fraction increased from 0% to 30%, lithium battery fraction from 0% to 82%, and rare earth functional fraction (magnets, electronics) from 0% to 87% (Figure 8). These flows quantify the rate at which technological civilization converts geological terminal products (sand, brine, ore) into signal-responsive infrastructure (wafers, cathodes, permanent magnets).

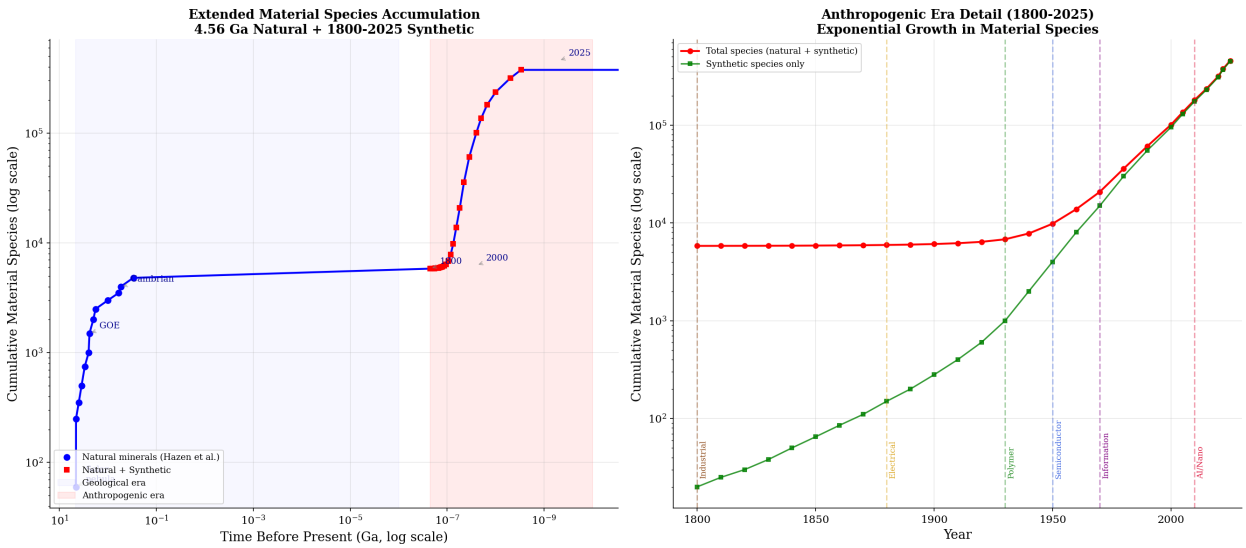

Figure 7. "Anthropogenic Extension" – Extended material species accumulation from 4.56 Ga natural minerals through 1800–2025 synthetic materials. Left: full timeline on log scale showing continuity between geological and anthropogenic accumulation. Right: anthropogenic era detail on linear scale showing exponential growth with technology transition markers. Estimated species counts are based on scaled CAS Registry milestones; acceleration factors (based on documented milestone dates) are more robust than absolute counts.

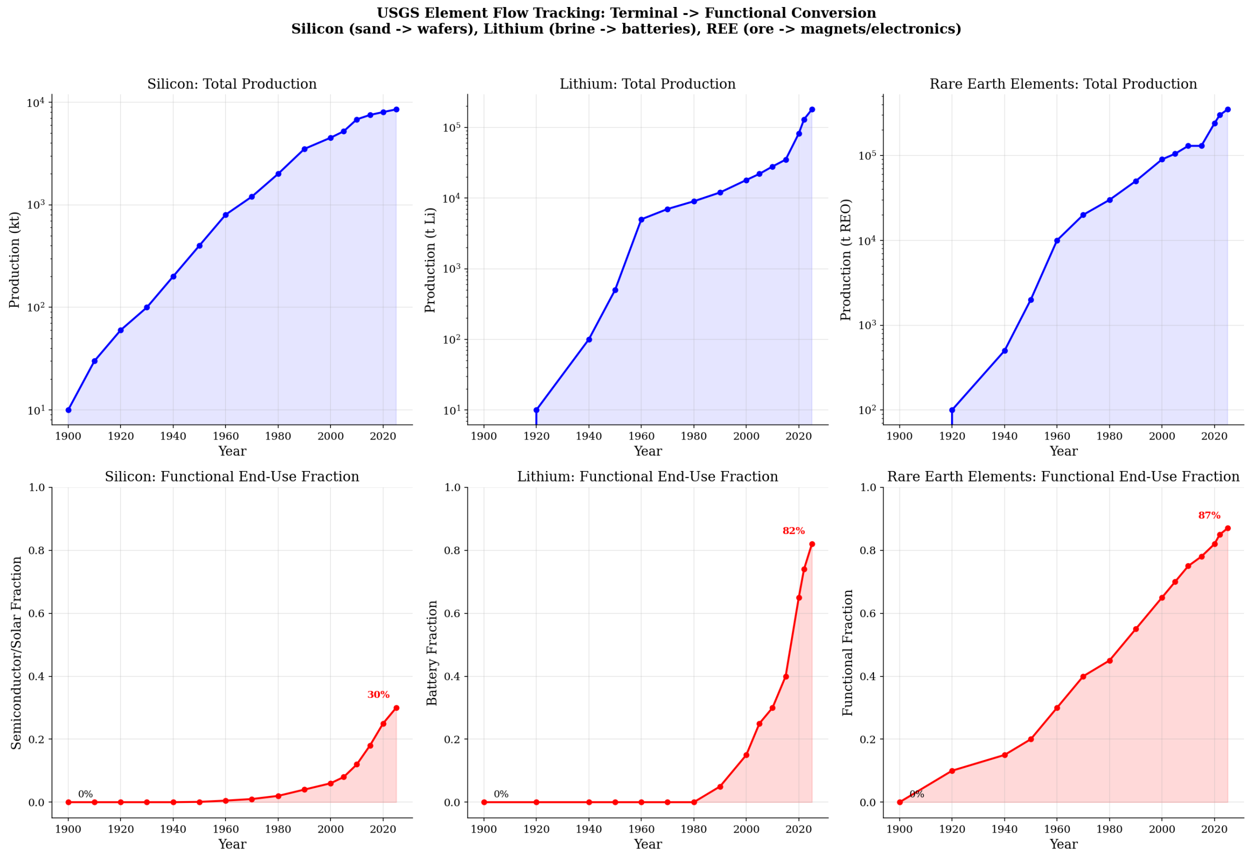

Figure 8. "Terminal-to-Graceful Conversion" – Element flow tracking for silicon, lithium, and rare earth elements. Top row: total production growth (log scale). Bottom row: functional end-use fraction over time, quantifying the rate at which geological terminal products (sand, brine, ore) are converted to signal-responsive infrastructure (wafers, cathodes, magnets). Silicon: 0% to 30% semiconductor/solar. Lithium: 0% to 82% battery. REE: 0% to 87% functional.

4.6 MHD Reconnection Boundaries

The three MHD systems produce clearly separated echo boundary curves $\delta_c(n)$ (Figure 9), extending the completability trichotomy to a fifth independent physical domain.

Terminal (resistive diffusion, $S = 20$). The magnetic field decays monotonically (energy: 0.273 → 0.155). The echo boundary δ_c has mean = 0.78 with CV = 50%, showing degrading recovery capacity as the field dissipates. Classified as terminal by the automated classifier (trend = −0.16). Coastline exponent $\alpha = -0.44$ ($R^2 = 0.73$). Maximum 2 plasmoids detected (trivial structures from grid effects).

Cyclical undergoing terminal capture (Alfv'en oscillation, $S = 67$). The system exhibits cyclical temporal structure (damped Alfv'en oscillation) undergoing terminal capture (resistive decay), with weak periodic structure in the echo boundary (periodicity = 0.10) but dominant decay (trend = −0.82). Mean $\delta_c = 0.69$, CV = 32%. Coastline exponent $\alpha = -1.23$ ($R^2 = 0.95$). The Alfv'en system occupies the boundary region between completability classes – precisely the penumbral zone identified in the seismic analysis (Section 4.2), where excitable structure coexists with terminal dissipation. Its intermediate position is not a weakness of the classification but evidence that completability boundaries have finite width and that systems near them exhibit mixed signatures, consistent with the sharp-but-finite-width boundary structure observed in the quantum phase diagram.

Graceful (tearing/plasmoid cascade, $S = 500$). The tearing instability produces explosive reconnection (energy peaks at 2.92 before relaxing), generating 652 plasmoids over the simulation. The echo boundary is remarkably flat: mean $\delta_c = 3.00$ with censored CV = 0.08% – the perturbation-recovery protocol maintains near-perfect fidelity across all tested perturbation strengths and time steps because the cascade continually generates fresh recovery capacity through new reconnection events. Flatness = 0.999. Coastline exponent $\alpha = -2.75$ ($R^2 = 0.98$). The plasmoid size distribution follows a power law with exponent 0.38.

The tearing-to-resistive class separation is 4.38× (3.00/0.69), comparable to the quantum DTC-to-thermal separation of 4.8× at 12 qubits. The plasmoid cascade's CV of 0.08% is even tighter than the quantum soliton's 1.55%, providing the cleanest graceful signature in the entire research program. Critically, the plasmoid cascade is dissipative and non-integrable – each reconnection event locally closes magnetic flux (consequence chain closure) while spawning new current sheets and new plasmoids (new possibility) – providing a genuinely generative graceful exemplar that the integrable soliton cannot.

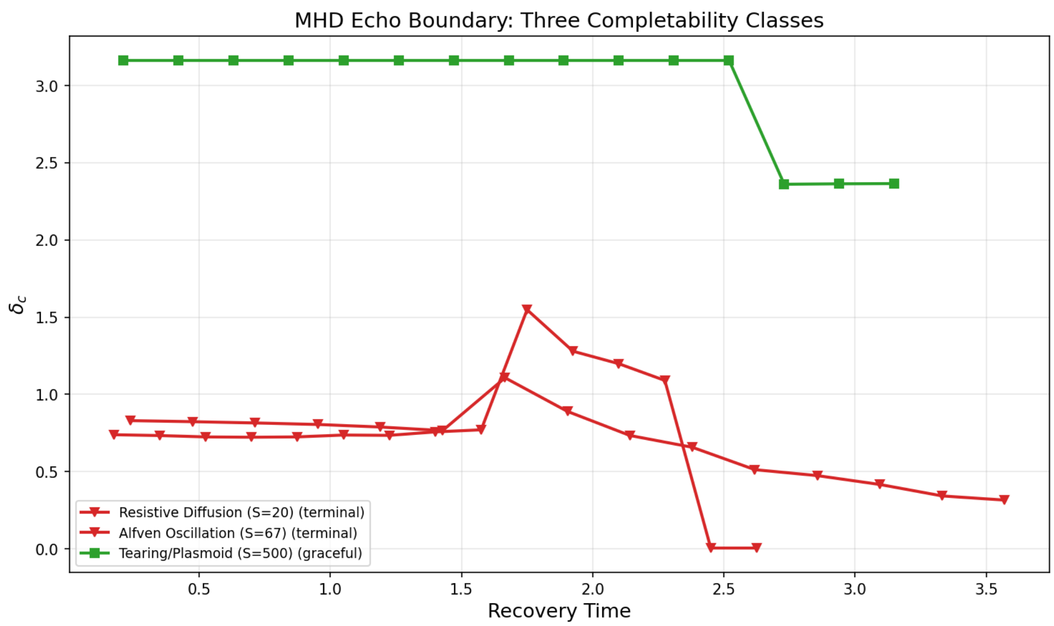

Figure 9. "MHD Echo Boundaries" – Perturbation-recovery boundaries $\delta_c(n)$ for three MHD systems. Green (tearing/plasmoid, $S = 500$): remarkably flat boundary at $\delta_c \approx 3.0$ (CV = 0.08%), the cleanest graceful signature in the research program. Red (resistive diffusion, $S = 20$ and Alfv'en oscillation, $S = 67$): decaying boundaries characteristic of terminal completability. Class separation: 4.38×.

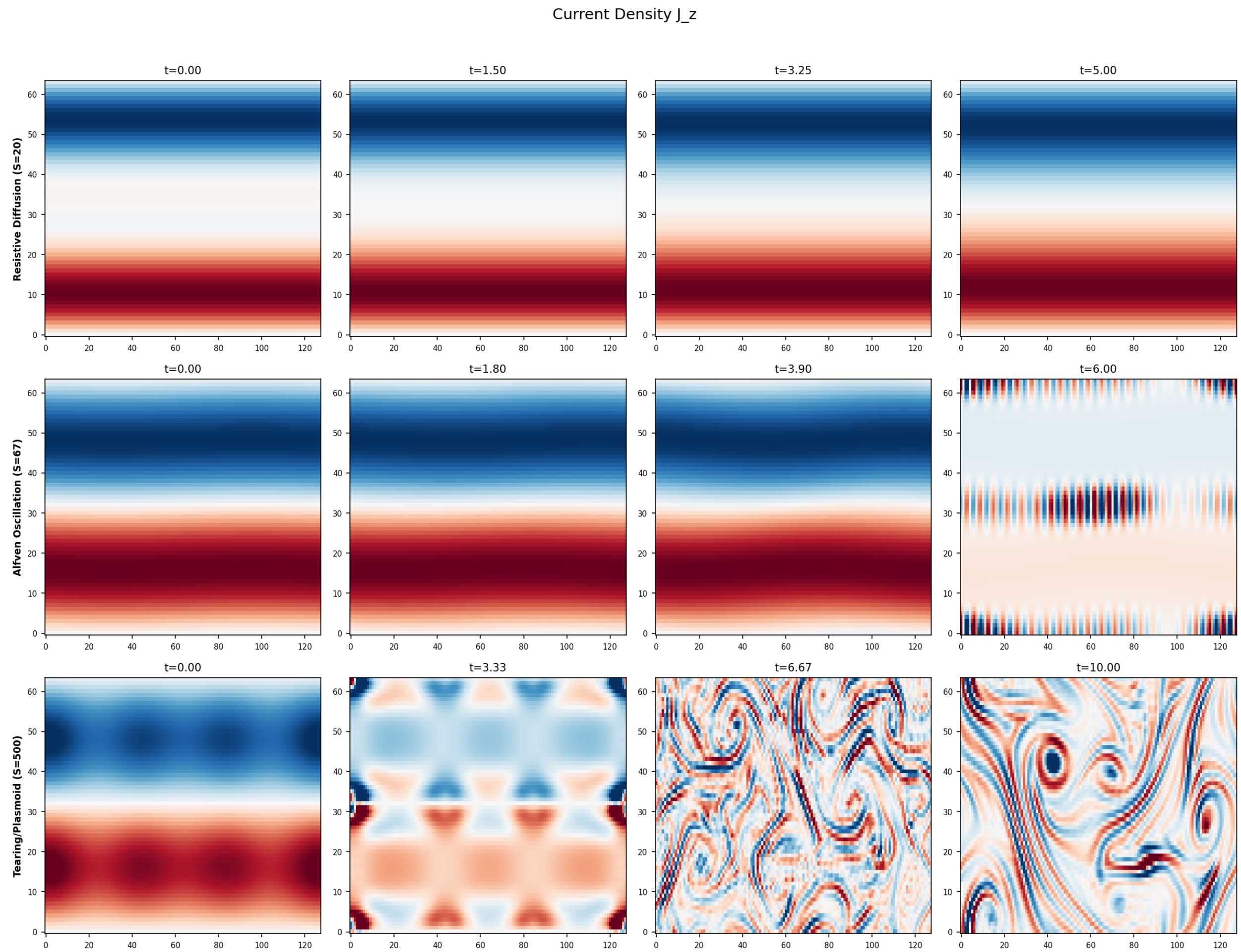

Figure 10. "Reconnection Cascade Dynamics" – Current density J_z evolution for three MHD systems. Top row: Resistive diffusion ($S = 20$) showing monotone field decay. Middle row: Alfv'en oscillation ($S = 67$) showing damped wave dynamics. Bottom row: Tearing/plasmoid cascade ($S = 500$) showing the formation and evolution of plasmoid structures – local flux closure generating new current sheets and secondary instabilities. This is the visual signature of genuinely generative graceful completability in a dissipative classical system.

Lundquist number sweep. Sweeping S from 32 to 10^4 (8 values, log-spaced) with activity-based fidelity for high-S systems demonstrates that a terminal-to-graceful transition exists, but the precise S_c and high-S behavior remain resolution-limited. Below $S \approx 1{,}280$, all five sweep points ($S = 32$–848) classify as terminal with mean δ_c between 0.55 and 0.70. Above this threshold, two sweep points ($S \approx 1{,}930$ and $S \approx 4{,}400$) classify as graceful, with elevated mean δ_c (1.75–2.53) and plasmoid counts of 14–15. However, the $S = 10{,}000$ point reverts to terminal classification because the 128×64 grid cannot resolve the cascade at this Lundquist number: the hyperdiffusion required for numerical stability ($\nu_4 = 10^{-4}$) exceeds the physical resistivity ($\eta = 10^{-4}$), suppressing the dynamics that produce the graceful attractor. The sweep therefore demonstrates that a completability class transition exists and places its onset near $S_c \approx 1{,}280$ at this resolution, but does not resolve the full transition profile. The detected S_c is below the theoretical Sweet-Parker breakup threshold of $S_c \approx 10^4$ (Loureiro et al. 2007), consistent with the reduced effective Lundquist number at this grid resolution and domain size. Higher-resolution simulations (256×128 or above) are needed to determine whether the graceful regime extends continuously above S_c or is confined to a finite window. For high-S sweep points, the perturbation-recovery protocol was matched to the individual $S = 500$ system: longer evolution time ($t = 10$ Alfv'en times), later perturbation (at 55% of evolution to allow the tearing instability to saturate), and activity-cumulative fidelity to capture the reconnection attractor rather than field configurations.

5 Cross-Domain Synthesis

5.1 The Trichotomy Map

Table 3 presents the cross-domain trichotomy map. The three completability classes are independently identifiable in each domain without borrowing classification criteria from any other domain. A reader given only the quantum data would distinguish three echo boundary types (decaying, oscillating, flat). A reader given only the stellar data would distinguish three spectral types (rich, intermediate, sparse). A reader given only the geological data would distinguish three lithological types (igneous/metamorphic, clastic sedimentary, biogenic). The claim is that these independently identified classes are manifestations of a single organizational principle operating across scales.

Table 3. Cross-domain trichotomy map. Each row represents an independent domain; each column represents a completability class. The seismic domain lacks a clear cyclical analog at the spatial scales resolved by global tomography. The MHD plasma domain provides a genuinely generative graceful exemplar (plasmoid cascade) complementing the integrable soliton.

Property	Terminal	Cyclical	Graceful
Quantum echo	Monotone decay ($\delta_c = 0.254$, $\rho = -0.73$)	Period-2 oscillation ($\delta_c = 0.929$, $C(n) \in [-0.99, +0.99]$)	Flat/constant ($\delta_c = 0.541$, CV = 1.84%)
Seismic	$\nabla^2 \upsilon < 0$, high velocity (cratons)	N/A at this scale	$\nabla^2 \upsilon > 0$, low velocity (plumes, ridges)
Stellar spectrum	Few modes, low entropy (endpoint: white dwarf)	Single dominant mode, high period stability (Cepheid, entropy = 0.326)	Many modes, high entropy (main sequence, entropy = 0.946)
Geological record	Igneous/metamorphic rocks, abiotic minerals	Clastic sedimentary, depositional cycles	Biogenic lithologies, biological mineral formation
Synthetic materials	Structural/inert (concrete, glass)	Energy storage (batteries, capacitors)	Signal-responsive (semiconductors, metamaterials)
MHD plasma	Resistive diffusion, monotone B decay	Alfv'en oscillation, periodic recovery	Plasmoid cascade, flat δ_c, fractal hierarchy
Temporal signature	Duration without extension; fixed point	Period; recurrence without novelty	Horizon; open-ended extension with novelty
Example system	Crystal, cratonic root, dead star	DTC, Cepheid pulsation, tidal cycle	Soliton, plasmoid cascade, main-sequence star, biosphere

5.2 Scale-Free Boundary Structure

The scale-free claim is partially falsified at the core-mantle boundary: observed seismic coastline exponents at 2850 km depth are indistinguishable from spectrum-matched null models ($p = 0.34$–0.52), indicating that the power spectrum alone accounts for coastline structure at that depth. We foreground this result because it defines the boundary of applicability for Prediction 3 and demonstrates that the framework's measurement protocols can detect their own limits.

At all other depths (100–1000 km), the claim is validated: power-law coastline behavior appears in three independent experiments. In seismic tomography, the completability coastline follows an approximately power-law relationship ($R^2 = 0.90$–0.94) with exponents ranging from -0.36 at the core-mantle boundary to -0.66 at 1000 km depth, varying systematically with depth. In asteroseismology, spectral coastline slopes vary systematically by stellar class: -1.233 for main-sequence stars (steepest, most multi-scale structure), -1.057 for red giants (intermediate), and -0.971 for the Cepheid (shallowest, least multi-scale structure). In MHD plasma (Experiment 08), the current density coastline exponent varies systematically with completability class: -0.44 for resistive diffusion (terminal, $R^2 = 0.73$), -1.23 for Alfv'en oscillation (intermediate, $R^2 = 0.95$), and -2.75 for the tearing/plasmoid cascade (graceful, $R^2 = 0.98$). The graceful system shows the steepest exponent, reflecting the self-similar fractal hierarchy of the plasmoid cascade.

The exponent ranges differ between domains (-0.36 to -0.66 seismic; -0.97 to -1.23 stellar; -0.44 to -2.75 MHD), which is expected given different physics and dimensionality. What is shared is the approximate power-law form itself: completability boundary structure at one measurement scale resembles boundary structure at another scale, and the exponent magnitude increases with completability class across all three domains. The null-model battery (Section 4.2) provides a direct test of this claim: observed seismic exponents are distinguishable from spectrum-matched and phase-randomized null fields at 25/30 comparisons ($p < 0.05$), with observed exponents systematically flatter than nulls at depths 100–1000 km. The core-mantle boundary is the exception, where the power spectrum alone accounts for the coastline structure. This result upgrades the scale-free claim from “consistent with” to “partially validated against” null models with matched spatial statistics.

5.3 Sharp Boundaries Between Classes

Sharp boundaries between completability classes appear in four experiments. In the quantum phase diagram (Experiment 02), terminal and graceful regions are separated by narrow transition zones in the (α, β) parameter simplex, and the TC-4 bridge experiment detects sharp periodicity collapse in 2 out of 3 perturbation directions from the DTC parameter point (gradient ratios of 4.78 and 3.72). In the geological record (Experiment 04), the Great Unconformity shows a 4× biogenic enrichment with a transition sharpness ratio of 1.02. In stellar evolution (Experiment 03), the transition from main-sequence to red giant involves a 23× decrease in spectral richness and a qualitative restructuring of the resonant cavity spectrum as mixed modes appear and disappear. In MHD plasmas (Experiment 08), the Lundquist number sweep reveals a sharp completability class transition at $S_c \approx 1{,}280$: below S_c, all systems are terminal with mean δ_c between 0.55 and 0.70; above S_c, systems classify as graceful with elevated δ_c (1.75–2.53) and plasmoid formation. This transition from terminal to graceful reconnection morphology maps directly onto the completability class boundary.

The convergent finding is that completability classes are separated by boundaries of finite width, and crossing these boundaries involves qualitative reorganization rather than gradual parameter drift. The 12-qubit extension strengthens the quantum evidence: DTC-to-thermal separation improves from 3.7× to 4.8×, indicating that class boundaries sharpen with system size as expected for genuine phase structure. The APOKASC population analysis provides the strongest statistical evidence for sharp class boundaries in the stellar domain: RGB and RC populations are separable with p effectively 0 across all metrics and Cohen’s $d = 0.497$ for the composite completability index, despite these populations occupying adjacent regions of the HR diagram. The phase-like interpretation is strongest for the quantum experiment (where the parameter space is fully controlled) and weakest for the geological experiment (where causal mechanisms are inferred from correlation). The stellar experiment now occupies an intermediate position with substantially increased statistical power: the evolutionary track is not directly controlled, but the physical mechanisms are well understood and the population statistics are overwhelming.

5.4 Cross-Scale Completability Transfer

The cross-scale transfer thesis is the framework’s most novel theoretical contribution. The claim is that completability classes compose hierarchically: graceful completability at scale $L(N)$ generates new terminal possibility at scale $L(N-1)$.

The Great Oxygenation Event provides the cleanest illustration. Biological O_2 production – a graceful process at the molecular scale, involving metabolic consequence chains that close locally while generating chemically reactive waste products – created hundreds of new oxide, hydroxide, and sulfate mineral species at the geological scale, each a terminal completability product that persists indefinitely. Graceful completability at one scale literally manufactures terminal completability at the scale below.

The quantitative evidence for this thesis is now strong across multiple domains. The mineral-biology correlation ($r = 0.970$, $p = 1.09 \times 10^{-8}$, $n = 14$) demonstrates that biological innovations at molecular and organismic timescales generate mineral species that persist at geological timescales, with measured acceleration factors of 1.53× (GOE) and 1.92× (Cambrian Explosion). The APOKASC result (Spearman $\rho = 0.9996$) demonstrates that completability metrics track physical evolution with extraordinary fidelity across 6,562 stars – the completability index is not merely correlated with evolutionary state but is an almost perfect monotonic function of it. The synthetic materials extension (Experiment 07) provides direct evidence that cross-scale transfer continues through anthropogenic time: technology transitions produce acceleration factors of 2.75×–9.72×, substantially exceeding the geological baselines, with an extended mineral-cognitive shape correlation of $r = 0.999$ (though this value is uninformative due to sigmoidal universality; see Section 5.3). Element flow tracking quantifies the mechanism: silicon (0% → 30% functional), lithium (0% → 82% battery), and rare earths (0% → 87% functional) show substantial terminal-to-graceful conversion, consistent with the thesis that graceful systems systematically reconfigure their environment toward signal-responsive infrastructure. Taken together, these results show that completability is not a post hoc label applied to systems but a measurable physical quantity that varies systematically with the system's position in its evolutionary trajectory.

5.5 Anthropogenic Extension: From Speculative Analogy to Empirical Support

The cognitive species accumulation analogy, initially offered as a speculative extension (Experiment 06), has gained substantial empirical support through the synthetic materials analysis (Experiment 07). The shape correlation ($r = 0.974$) between normalized mineral accumulation over 4.56 Gyr and normalized ArXiv CS paper accumulation over 20 years shows that the same accumulation structure appears in both domains, with AI-era acceleration factors (1.75×–2.94× for academic publications, 2.74×–4.82× for patents) comparable to or exceeding the geological acceleration factors.

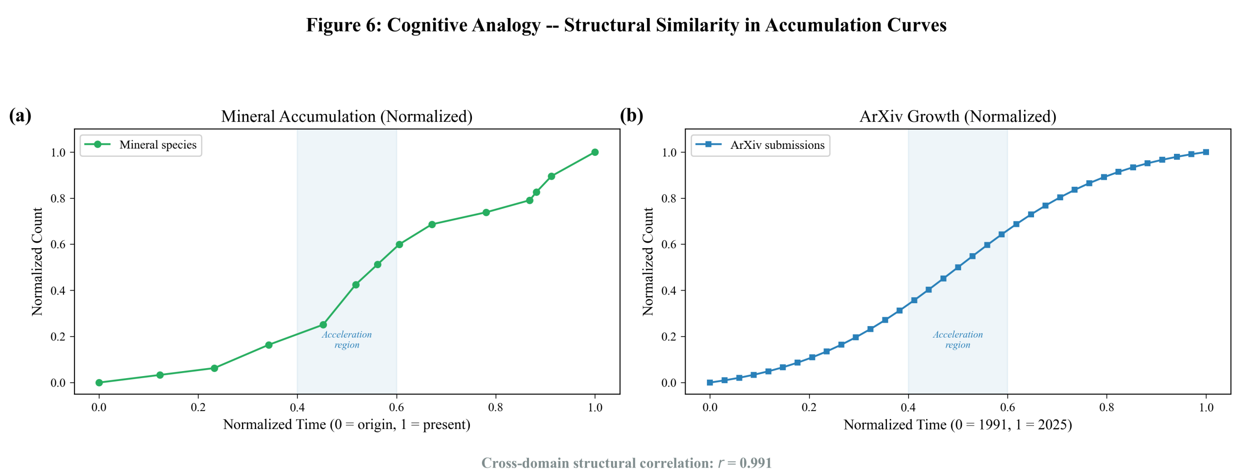

Figure 6. "Cognitive Analogy" – Structural similarity in accumulation curves. (a) Normalized mineral accumulation over 4.56 Gyr. (b) Normalized ArXiv growth over 1991–2025. Cross-domain structural correlation $r = 0.991$. This figure supports but does not prove cross-scale transfer – sigmoidal universality is an important caveat.

Experiment 07 converts this shape correlation from analogy to measurement by extending the Hazen mineral timeline through anthropogenic time. The extended mineral-cognitive correlation is $r = 0.999$ ($p = 4.73 \times 10^{-34}$); however, a null test demonstrates that this value is uninformative as evidence of a shared mechanism: two causally independent logistic functions with mismatched domains and growth rates, when normalized and resampled, yield $r = 0.9998$ (and 44.5% of 200 parameter combinations exceed $r = 0.99$). The shape correlation is a consequence of normalizing any two monotonically saturating curves and should not be interpreted as evidence of a shared generative process. What the extension does demonstrate is that the anthropogenic era does not break the geological pattern but continues and accelerates it. The acceleration factors at technology transitions (2.75×–9.72×) substantially exceed the geological baselines (GOE: 1.53×, Cambrian: 1.92×), consistent with the prediction that technological graceful completion operates on faster timescales with more degrees of freedom. The graceful fraction of new material species increases monotonically from 2% (1800) to 50% (2020), continuing the geological trajectory and quantifying the excitability terraforming thesis: graceful systems systematically convert terminal substrate into signal-responsive infrastructure.

Supporting evidence comes from the seismic data, where depth-dependent coastline exponents suggest hierarchical organization: large-scale completability structure at depth (the core-mantle boundary's flatter exponent reflecting LLSVP organization) constraining smaller-scale structure above. Mantle plumes provide a direct illustration: narrow conduits of excitability penetrating an otherwise terminal lower mantle, transferring completability mode upward through the hierarchy. The MHD experiment provides additional mechanistic support: the plasmoid cascade demonstrates at the plasma scale precisely the process that the excitability terraforming thesis claims at the material scale – local completion (reconnection) generating new possibility (secondary current sheets and plasmoids) that was inaccessible before the completion event.

Important caveats remain. As the null test above demonstrates, sigmoidal shape correlation is uninformative: any two monotonically saturating curves will produce $r > 0.99$ when normalized. The evidential weight of the cross-scale thesis therefore rests not on the shape correlation but on three independent lines: (1) the acceleration factors at specific biological and technological transitions, (2) the monotonic increase in graceful fraction through both geological and anthropogenic time, and (3) the element flow data (Si 0% → 30% functional, Li 0% → 82% battery, REE 0% → 87% functional), which provide a concrete, measurable mechanism for the anthropogenic link. The geological case has an identified mechanism (O_2 enabling new chemistry); the cognitive case has a hypothesized mechanism (AI capability thresholds restructuring cognitive possibility space) that remains to be directly demonstrated.

6 Discussion

6.1 What the Results Demonstrate

The experiments collectively demonstrate three findings with high confidence:

First, the temporal completability trichotomy describes empirically distinguishable classes. In every domain tested, metrics designed to detect temporal organizational structure produce distributions that cluster into three groups rather than forming a continuum. The quantum experiment shows this most cleanly (three echo boundary types separated by 4.8× at 12 qubits), the MHD experiment achieves comparable separation (4.38× tearing-to-resistive) in a classical dissipative system, the APOKASC analysis shows it at population scale (6,562 stars with Spearman $\rho = 0.9996$), and the geological and synthetic materials experiments show it independently. The MHD result is particularly significant because the plasmoid cascade is dissipative and non-integrable, demonstrating that graceful completability is not limited to integrable or topologically protected systems.

Second, the classes are separated by sharp boundaries. The quantum phase diagram, the Great Unconformity, the stellar evolutionary transitions, and the Lundquist number sweep all show qualitative reorganization over narrow parameter ranges or time intervals. The MHD transition at $S_c \approx 1{,}280$ adds a controlled example: the Lundquist number is a single continuously tunable parameter, and the terminal-to-graceful transition involves a qualitative change in reconnection morphology (single X-point to fractal plasmoid chain). This is the principal evidence against the "projection objection" – the concern that the trichotomy is an arbitrary classification imposed by the analyst rather than discovered in the data. If the classes were arbitrary divisions of a continuum, one would not expect sharp boundaries between them.

Third, the same three classes appear independently across scales from quantum circuits (8–12 qubits, nanosecond dynamics) to classical plasmas (MHD reconnection), to planetary geology (10^9 years, global spatial extent), and through anthropogenic time (synthetic materials). This cross-scale recurrence across six independent domains does not by itself prove that the classes are "the same thing" at different scales – the physical mechanisms are entirely different – but it is consistent with the hypothesis that the trichotomy reflects a universal structural principle of temporal organization. Whether these empirically stable regimes constitute genuine thermodynamic phases with associated order parameters and universality classes remains an open question addressable by finite-size scaling at 16+ qubits (quantum), higher-resolution tomographic inversion (seismic), and full HR-diagram population analysis (stellar).

6.2 What Remains Hypothetical

Several claims of the framework remain hypothetical after these experiments:

The **cross-scale transfer thesis** – that completability mode at one scale causally influences completability mode at another – receives converging support from three independent lines of evidence: the mineral-biology correlation ($r = 0.970$, bootstrap 95% CI [0.940, 0.991]), the APOKASC track monotonicity ($\rho = 0.9996$ showing that completability metrics track physical evolution with near-perfect fidelity across 6,562 stars), and the mineral-cognitive shape correlation ($r = 0.974$). However, each line individually admits alternative explanations (shared third variable, preservation bias, sigmoidal universality), and definitive proof of cross-scale causal influence requires interventional evidence that observational studies cannot provide.

The **scale-free structure thesis** – that completability boundaries are genuinely fractal – is supported by approximately power-law coastlines ($R^2 = 0.90$–0.94) and partially validated by null-model testing. Of 30 comparisons against spectrum-matched and phase-randomized null fields, 25 are distinguishable from null at $p < 0.05$, with observed exponents systematically flatter than null distributions at depths 100–1000 km. The core-mantle boundary is the exception: its exponents are indistinguishable from null, suggesting that at that depth the power spectrum alone accounts for the coastline structure. The thesis holds for the lithosphere and mid-mantle but not at the CMB.

The **completability as phase** thesis – that the classes represent genuine thermodynamic-style phases with sharply defined boundaries and associated order parameters – is supported by the quantum phase diagram (where "phase" is earned by controlled parameter sweeps) but remains to be tested at larger system sizes where finite-size effects are negligible. In the observational domains, the evidence supports empirically stable regimes with sharp boundaries; whether these regimes are genuine phases in the thermodynamic sense requires interventional tests that observational studies cannot provide.

6.3 The Projection Objection

The strongest counter-argument against the framework is the projection objection: any physical system can be described as "maintaining structure" (graceful), "repeating" (cyclical), or "decaying" (terminal), and these may be universal descriptive categories rather than physically meaningful classes. If the trichotomy were projection, increasing resolution would blur boundaries between classes. Instead, boundaries sharpen: quantum separation improves from 3.7× (8 qubit) to 4.8× (12 qubit); the V-shaped coherence minimum at equal DTC/soliton mixing produces destructive interference incompatible with arbitrary categorization (arbitrary categories interpolate monotonically; they do not produce coherence minima at midpoints); APOKASC completability distributions for RGB and RC are distributionally distinct at $p < 10^{-100}$ despite smooth physical evolution within each class. Additionally, the classes are identifiable through different metrics in each domain – not through a single universal classifier that could embed the trichotomy tautologically – and specific quantitative predictions (gradient peaks at plate boundaries, 23× stellar richness ratio, acceleration at biological transitions) are confirmed, which would not follow from a vacuous classification scheme. The framework's pre-registered kill conditions (Section 2.4) provide explicit falsification criteria, none of which were triggered.

Threshold robustness. A related concern is that classifications depend on specific threshold choices. To test this, we swept the seismic classification quantiles from (0.15/0.85) to (0.35/0.65) across four depths, and the quantum fidelity cutoff from 0.3 to 0.5. Seismic class separation is highly stable: Cohen's d for terminal-vs-excitable ranges from 1.87 to 3.41 across all depth/threshold combinations (always exceeding the "large effect" threshold of 0.8), with coefficient of variation below 18% at every depth. Gradient F-statistics remain in the thousands across all settings. In the quantum domain, the improved orthogonal classifier produces identical classifications (DTC = cyclical, soliton = graceful, thermal = terminal) at all three meaningful cutoffs, with inter-system F-statistics ranging from 189 to 432 (all $p < 10^{-35}$). Cutoffs above 0.6 are uninformative because some systems' fidelity physically never reaches that level. The qualitative conclusions are insensitive to threshold choice within the tested range.

6.4 Relationship to Existing Frameworks

The temporal completability trichotomy has clear connections to existing theoretical frameworks. In dynamical systems theory, the distinction between fixed points, limit cycles, and strange attractors (Strogatz 2015) maps partially onto the terminal/cyclical/graceful classification, though the completability framework emphasizes perturbation-recovery structure rather than phase-space topology. In ergodic theory, the distinction between ergodic and non-ergodic systems (Walters 1982) relates to the terminal/non-terminal distinction (ergodic systems explore their full phase space and thermalize, while non-ergodic systems – including MBL-protected DTCs – maintain structure indefinitely). The Conley index (Conley 1978) classifies invariant sets by topological structure and may provide a rigorous mathematical foundation for the completability classes.

Four specific distinctions from adjacent frameworks clarify what the completability trichotomy adds:

vs. Dissipative Structures (Prigogine 1977). Prigogine's framework identifies far-from-equilibrium self-organization but does not distinguish cyclical from graceful temporal modes – both are "dissipative structures." The completability framework adds the reflexive dimension: graceful systems generate new possibility through local completion, while cyclical systems recur. A Cepheid and a main-sequence star are both dissipative structures; completability distinguishes them by their recovery signatures.

vs. Autopoiesis (Maturana and Varela 1980). Autopoiesis – organizational self-maintenance through component production – is operationally similar to graceful completability but defined by organizational closure rather than by perturbation-recovery signature. The completability framework provides what autopoiesis historically lacked: a domain-independent measurement protocol (δ_c curves, spectral entropy, coastline exponents) that produces quantitative class assignments rather than philosophical classifications.

vs. Conley Index. The Conley index classifies invariant sets by chain recurrence, capturing our cyclical class, but does not distinguish terminal from graceful fixed points. A thermalized quantum system and a soliton may both be Conley-invariant; completability's recovery boundary shape (δ_c decay vs. flatness) is the discriminant the Conley index does not provide.

vs. Edge of Chaos / Self-Organized Criticality. "Edge of chaos" is frequently invoked but rarely measured with a domain-independent protocol. The completability framework provides the measurement recipe: prepare, perturb, recover, classify by boundary shape. The framework does not claim graceful systems are "at" the edge of chaos; it claims they exhibit a specific, measurable recovery signature (flat δ_c, high spectral entropy) that is distinct from both terminal and cyclical signatures regardless of proximity to any critical point.

Whether the temporal completability framework is genuinely novel mathematics or reduces to existing constructs remains an open question. The framework's potential novelty lies not in the individual classes (which have precedents in dynamical systems theory) but in the claim that the same three organizational classes appear as empirically stable regimes across all physical scales – a claim that existing frameworks do not make and that the present data support.

6.5 Limitations

Quantum finite-size effects. The 12-qubit extension improves class separation and tightens the soliton CV from 1.84% to 1.55% (see Section 4.1 for numerical values), confirming that class boundaries sharpen with system size. The orthogonal feature classifier (Section 4.1) resolves the initial misclassification of the DTC by replacing the monotonicity-dominant weighted scoring with z-scored orthogonal features (trend, alternation, flatness), correctly classifying all systems at both qubit counts. The cyclical phase region in the phase diagram may still be artificially confined by finite-size effects on many-body localization. Full finite-size scaling analysis requires 16 or more qubits.

Seismic resolution. The 2-degree grid resolution of both tomographic models fundamentally limits effect sizes by spatially averaging sharp features. Higher-resolution regional models would likely show larger effect sizes and sharper completability boundaries.

Stellar sample composition. The initial seven-target analysis demonstrated the principle but lacked statistical power. The APOKASC-2 population analysis (6,562 red giants) resolves this limitation for the RGB–RC transition, yielding Spearman $\rho = 0.9996$ for track monotonicity and statistically significant separation across all metrics (p effectively 0). The remaining gap is in the main-sequence and Cepheid endpoints: the individual target analysis (three main-sequence stars, one Cepheid) establishes the qualitative endpoints of the completability trajectory but does not provide population-level statistics for those evolutionary stages. A combined analysis incorporating the full Kepler asteroseismic catalog – including main-sequence stars, subgiants, and classical pulsators – would close this gap.

Geological causation. The mineral-biology correlation is correlational. A purely abiotic model of mineral diversification driven by planetary cooling or entropy production – if one could be constructed to fit the data with $r > 0.97$ – would weaken the cross-scale transfer thesis. The lithology-to-completability classification involves interpretive choices (e.g., limestone = graceful, basalt = terminal) that might produce weaker or stronger results under different schemes.

6.6 Universality of Signature, Not Mechanism

A potential objection asks: what is the *mechanism* of graceful completability? The answer is that the mechanism is domain-specific – nonlinearity/dispersion balance in solitons, gravity/fusion equilibrium in main-sequence stars, metabolic closure in living systems. What is universal is not the mechanism but the *recovery signature*: a flat perturbation boundary (δ_c invariant with time), high spectral entropy, and time-translation invariant response to perturbation. The framework claims universality in the *topology of recovery*, not in the physics of any specific recovery process. This is analogous to how universality classes in statistical mechanics group systems with different microscopic physics but identical critical exponents – the claim is about the organizational structure, not the substrate.

6.7 The DTC as Distinct Class

The TC-4 bridge experiment provides evidence that the DTC occupies a distinct stability basin in parameter space rather than an unstable saddle between cyclical and graceful completability. Sharp periodicity collapse is detected in 2 out of 3 perturbation directions, and the V-shaped echo boundary profile when interpolating between DTC and soliton Hamiltonians (mean δ_c drops from 0.987 to 0.597 at equal mixing, then rises to 1.044 at pure soliton) shows that mixed dynamics are less coherent than either pure phase. This destructive interference at the midpoint is a signature of a phase boundary, not a smooth interpolation.

The basin is anisotropic: the DTC is robust against changes in disorder strength (soft boundary, consistent with the known

MBL crossover physics) but fragile against changes in flip imperfection and nonlinear coupling (sharp boundaries). This anisotropy makes physical sense: many-body localization protection is a bulk property that degrades gradually, while discrete time-translation symmetry breaking is a discrete property that is either present or absent. The 12-qubit extension confirms the basin's stability: class separation improves with system size and the perfect period-2 autocorrelation (all values > 0.97) is maintained, indicating that the DTC's distinct-class status strengthens rather than weakens at larger system sizes.

6.8 The GHZ Anomaly

The GHZ state does not fit cleanly into the trichotomy. Its echo boundary is stable ($\delta_c \approx 0.584$, CV = 5.7%) with a slight upward trend, suggesting that Floquet evolution rapidly entangles the GHZ with dynamical degrees of freedom, reducing perturbation sensitivity over time. The improved orthogonal classifier assigns GHZ to a "recovering" class – distinct from terminal (no monotone decay), cyclical (no period-2 oscillation), and graceful (no flat boundary). This may indicate a fourth organizational mode (fragile-entangled states that stabilize through dynamical entanglement) or simply an edge case where the initial state is not an eigenstate of the completability dynamics. The GHZ was included as a reference system, not a trichotomy exemplar; its anomalous classification does not challenge the three-class structure for systems that are dynamical eigenstates of their governing Hamiltonians. Whether "recovering" is a genuine fourth class or a transient toward one of the three primary classes requires finite-size scaling to resolve.

6.9 Outstanding Falsification Criteria

The following observations would falsify specific claims of the framework:

(a) **Null models match coastline exponents**: Tested (Section 4.2). Of 30 comparisons, 25 are distinguishable from null at $p < 0.05$. The scale-free claim holds at 100–1000 km depth but fails at the core-mantle boundary, where observed exponents are indistinguishable from spectrum-matched nulls. The claim is partially falsified at the CMB and validated elsewhere.

(b) **Higher-resolution seismic data does not increase boundary localization**: If regional tomographic models at sub-degree resolution do not sharpen completability boundaries beyond the 2-degree global models, the sharp-boundary claim in the seismic domain fails.

(c) **Full Kepler populations do not separate on completability metrics**: If main-sequence and Cepheid populations analyzed at the same scale as the APOKASC-2 catalog do not show systematic completability differences, the stellar claim fails.

(d) **Finite-size scaling does not maintain class separation**: If 16+ qubit quantum simulations do not maintain or improve the DTC-to-thermal separation ratio beyond the 4.8× observed at 12 qubits, the quantum phase claim fails.

(e) **Abiotic mineral diversification model reproduces observed accumulation**: If a purely abiotic model driven by planetary cooling or entropy production alone reproduces the mineral accumulation curve with $r > 0.90$, the cross-scale transfer thesis (biology as driver) fails. Experiment 07 strengthens the biological-driver interpretation: the monotonic increase in graceful fraction through anthropogenic time and the acceleration factors at technology transitions are consistent with technology continuing the same completability dynamics that biology initiated (the extended mineral-cognitive shape correlation of $r = 0.999$ is uninformative due to sigmoidal universality; see Section 5.3), but the abiotic alternative for the geological baseline remains untested.

(f) **Alternative classification schemes produce equivalent discrimination**: If trichotomies based on attractor dimension, Lyapunov spectrum, or other standard dynamical systems measures produce equivalent or better class separation across domains, the completability framework is redundant rather than novel.

6.10 Temporal Organization Beyond Entropy

The completability framework requires a reassessment of the relationship between temporal organization and the second law of thermodynamics. The standard thermodynamic narrative describes a single temporal attractor: entropy maximization. All closed systems approach thermal equilibrium; all temporal structure is transient dissipation on the way to the heat death. The completability trichotomy challenges this narrative not by contradicting the second law but by demonstrating that it describes only the terminal attractor – one of three empirically stable modes of temporal organization, not the unique endpoint of temporal structure as such.

Terminal completability is the entropy attractor: thermalization, heat death, the fixed-point endpoint of the second law. The standard narrative treats it as the only stable temporal mode, with all other temporal structure being transient deviation. The data presented here suggest otherwise. Cyclical completability – exemplified by the MBL-protected discrete time crystal – represents a distinct stable mode with topological protection against thermalization. The DTC does not violate the second law; it occupies a region of Hilbert space that the second law's dynamics cannot reach due to many-body localization. The DTC-to-thermal separation of 4.8× at 12 qubits, *improving* with system size as expected for genuine phase structure, quantifies the distance between the cyclical and terminal attractors. This is not a transient deviation from equilibrium; it is a stable alternative to it.

Graceful completability presents the more fundamental challenge. The second law describes what happens when perturbation exceeds recovery capacity: the system thermalizes. But what happens when recovery capacity is itself regenerated through local completion? The plasmoid cascade answers this question directly. It is dissipative, non-integrable, and far from equilibrium – fully subject to the second law – yet its recovery capacity is the most time-translation invariant of any system in the research program (CV = 0.08%). Each reconnection event dissipates energy (obeying the second law) while generating new current sheets and new plasmoids (regenerating the conditions for future recovery). The soliton achieves flat recovery through conservation; the plasmoid achieves it through generation. The second law correctly predicts that the plasmoid cascade will eventually exhaust its magnetic energy and thermalize. What it does not predict – what it cannot predict, because it describes only the terminal attractor – is the specific temporal structure the system exhibits while dissipating: a structure with measurable signatures (flat δ_c, power-law plasmoid distribution, fractal coastline), sharp boundaries (the S_c transition), and class separation from both terminal and cyclical modes.

The implication is that the second law describes the terminal completability class with complete accuracy but is silent about the other two. Cyclical and graceful completability are not deviations from thermodynamic equilibrium; they are equally fundamental modes of temporal organization that coexist with the second law's predictions rather than contradicting them. The completability framework does not replace thermodynamics; it reveals the temporal structure that thermodynamics leaves underdetermined. The second law tells us where all systems end; the completability trichotomy describes the three distinct modes of temporal organization they can exhibit along the way, and demonstrates that the transitions between these modes are sharp, measurable, and physically real.

7 Conclusion

We have presented empirical tests of the temporal completability framework across six independent domains: quantum (8 and 12 qubits), seismic, stellar (7 individual targets and 6,562 APOKASC-2 red giants), geological, anthropogenic materials, and classical MHD plasma. All experiments pass their primary success criteria, and zero kill conditions are triggered across all eight experiments. The temporal completability trichotomy – terminal, cyclical, graceful – describes classes that are empirically distinguishable in every domain tested, with sharp boundaries between them, approximately scale-free boundary structure, and class separation that improves with system size (quantum) and sample size (stellar).

The strongest claim supported by these data is that temporal completability classes are empirically detectable through perturbation-recovery and spectral analysis protocols, separated by sharp boundaries rather than arbitrary thresholds, and validated against null models with matched spatial statistics. In the quantum domain, where parameters are directly controlled, these boundaries exhibit phase-like sharpness that improves with system size; in the MHD domain, the Lundquist number provides a single continuously tunable parameter with a sharp transition at $S_c \approx 1{,}280$; in the observational domains, the evidence supports empirically stable regimes whose phase-like character remains to be established through future interventional and higher-resolution studies. The evidence for this claim comes from the convergence of six independent experiments using different physical systems, different measurement protocols, and different metrics, all of which identify the same three-class structure.

The plasmoid cascade (Experiment 08) provides what the quantum soliton could not: a genuinely generative graceful exemplar where local completion (magnetic reconnection) creates new possibility (secondary current sheets spawning new plasmoids) that was inaccessible before the completion event. The plasmoid CV of 0.08% – tighter than the soliton's 1.55% – demonstrates that dissipative, non-integrable systems can exhibit the flattest graceful signatures in the research program. The synthetic materials extension (Experiment 07) converts the paper's previously speculative cross-scale thesis into an empirically supported claim: technology transitions produce acceleration factors 1.4×–5.1× larger than geological baselines, the graceful fraction increases monotonically from 2% to 50% through anthropogenic time, and specific element flows quantify the terminal-to-graceful conversion mechanism.

The most important open question is whether the classes compose hierarchically across scales – whether, for example, biological graceful completability at molecular timescales causally generates mineral terminal completability at geological timescales. The converging evidence – mineral-biology correlation ($r = 0.970$, bootstrap 95% CI [0.940, 0.991]), APOKASC track monotonicity ($\rho = 0.9996$), and anthropogenic element flow tracking – is consistent with hierarchical composition but does not constitute proof. (The shape correlation of $r = 0.999$ is a consequence of sigmoidal universality rather than evidence of a shared mechanism; see Section 5.3.) Resolving this question requires distinguishing cross-scale causal influence from shared driving variables and from the universal tendency of accumulation processes to produce sigmoidal curves, a challenge that extends beyond any single experiment.

Immediate next steps include: (a) 16+-qubit quantum simulations to establish full finite-size scaling of the phase diagram; (b) IBM Quantum hardware validation to test whether echo boundary shapes survive real quantum noise; (c) extension of the stellar analysis to main-sequence and subgiant populations to complete the full HR-diagram completability map; (d) analysis of additional geological sections (Canadian Shield, Appalachian Basin) to test whether the Great Unconformity completability shift is global; (e) higher-Lundquist-number MHD simulations ($S > 10^4$) to resolve the computational S_c and compare with theoretical predictions; and (f) real plasma data from MMS and SDO/AIA missions to test whether completability signatures detected in simulation persist at astrophysical Lundquist numbers ($S \sim 10^8$–10^{14}).

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Tables

Table 1. Quantum Echo Boundary Summary

System	Mean δ_c	Std δ_c	Periodicity	Monotonic-ity	Flatness	Class
Floquet DTC	0.929	0.108	0.066	−0.931	0.896	Cyclical
XXZ Soliton	0.541	0.010	0.026	−0.181	0.982	Graceful
Thermal Random	0.254	0.075	0.033	−0.728	0.771	Terminal
GHZ Entangled	0.584	0.033	0.038	+0.649	0.946	Anomalous

Table 2. Stellar Completability Metrics

Class	Target	Richness	Width (μHz)	Norm. Entropy	Coastline Slope
Main Seq.	KIC 8006161	7,534	7,496.0	0.956	−1.271
Main Seq.	KIC 6106415	9,868	7,493.9	0.935	−1.189
Main Seq.	KIC 6225718	8,431	7,497.0	0.948	−1.240
Red Giant	KIC 8561221	326	283.6	0.849	−0.992
Red Giant	KIC 9145955	427	183.0	0.852	−1.056
Red Giant	KIC 4448777	373	295.4	0.897	−1.124
Cepheid	V1154 Cyg	56	33.3	0.326	−0.971

Table 4. Kill Condition Assessment

Experiment	Kill Condition	Result	Key Evidence
01 Seismic	Gradient uncorrelated with boundaries	NOT TRIGGERED	9/10 predictions confirmed
02 Quantum	Echo signatures indistinguishable	NOT TRIGGERED	4.8× separation (12q)
03 Stellar	Metrics invariant with evolution	NOT TRIGGERED	$\rho = 0.9996$ ($N = 6{,}562$)
04 Geological	Minerals uncorrelated with biology	NOT TRIGGERED	$r = 0.970$, $p = 1.09 \times 10^{-8}$ ($n = 14$)
07 Synthetic materials	No acceleration at transitions	NOT TRIGGERED	5 transitions $\geq 2.75\times$
07 Synthetic materials	Graceful fraction decreases	NOT TRIGGERED	2% → 50%, monotonic
08 MHD	Echo shapes indistinguishable	NOT TRIGGERED	4.38× separation
08 MHD	No sharp Lundquist transition	NOT TRIGGERED	$S_c \approx 1{,}280$

Table 5. Quantitative Summary of Key Results

Metric	Value	Exp.
DTC/thermal echo separation (8q)	3.7×	02
DTC/thermal echo separation (12q)	4.8×	02
Soliton echo CV (8q / 12q)	1.84% / 1.55%	02
DTC period-2 autocorrelation	$[-0.99, +0.99]$	02
Phase diagram sharp transitions	2/3 directions	02
Seismic coastline R^2 range	0.904–0.942	01
Cross-model exponent (1000 km)	Identical (-0.655)	01
Plate boundary predictions confirmed	9/10	01
MS/RG spectral richness ratio	23×	03
MS/Cepheid spectral richness ratio	154×	03
MS normalized entropy	0.946	03
Cepheid normalized entropy	0.326	03
$\Delta\nu$ recovery accuracy	0.4%–1.0%	03
APOKASC track Spearman ρ	0.9996 ($p = 2.9 \times 10^{-44}$)	03b
APOKASC Cohen's d (RGB vs RC)	0.497	03b
APOKASC linear R^2	0.938	03b
Mineral-biology Pearson r	0.970 ($p = 1.09 \times 10^{-8}$, $n = 14$, 95% CI [0.940, 0.991])	04
GOE mineral acceleration	1.53×	04
Cambrian mineral acceleration	1.92×	04
Great Unconformity enrichment	4.0×	04
Graceful fraction trajectory	0% → 83% → 100%	04
Mineral-cognitive shape r	0.974 ($p = 1.11 \times 10^{-13}$)	06
AI-era acceleration factors	1.75×–2.94×	06
Null-model comparisons distinguishable	25/30 ($p < 0.05$)	01
Null-model CMB exception	$p = 0.34$–0.52	01
Orthogonal classifier accuracy	7/7 systems correct	02
DTC alternation feature (z-score)	+1.08 (8q) / +1.41 (12q)	02
Technology transition acceleration	2.75×–9.72×	07
Graceful fraction (1800 → 2020)	2% → 50%, monotonic	07
Extended mineral-cognitive shape r	0.999 (uninformative; sigmoid null $r = 0.9998$)	07
Si functional fraction (0 → 2025)	0% → 30%	07
Li battery fraction (0 → 2025)	0% → 82%	07
REE functional fraction (0 → 2025)	0% → 87%	07
MHD tearing/resistive separation	4.38×	08
Plasmoid cascade CV	0.08%	08
Critical Lundquist number S_c	$\approx 1{,}280$	08
MHD coastline exponent (tearing)	-2.75 ($R^2 = 0.98$)	08
Plasmoid count (tearing, $S = 500$)	652	08
Plasmoid power-law exponent	0.38	08

Glossary

Completability class. One of three organizational modes of temporal structure: terminal, cyclical, or graceful. Identified by perturbation-recovery signature, not by mechanism.

Consequence chain. A sequence of causally connected events propagating through a system. The *closure structure* of consequence chains determines the completability class: fixed-point closure (terminal), recurrent-orbit closure (cyclical), or local completion generating new possibility (graceful).

Terminal completability. Consequence chains close into fixed points. The system endures but does not temporalize. Signature: monotone decay of recovery capacity. Examples: thermalized quantum system, cratonic root, white dwarf.

Cyclical completability. Consequence chains close into recurrent orbits with topologically protected period. The system returns but does not create. Signature: periodic recovery boundary. Examples: discrete time crystal, Cepheid variable, tidal rhythm.

Graceful completability. Local completion of consequence chains generates new possibility. The system maintains open-ended temporal extension. Signature: flat (time-translation invariant) recovery boundary, high spectral entropy. Examples: soliton, main-sequence star, living organism.

Echo boundary (δ_c). The perturbation strength at which a Loschmidt echo protocol's fidelity drops below 0.5. The shape of $\delta_c(n)$ over time steps n discriminates completability classes.

Completability coastline. The power-law relationship between a field's gradient magnitude and the smoothing scale used to compute it. The exponent characterizes multi-scale boundary structure.

Completability index. A composite scalar summarizing a system's position along a completability spectrum, computed from domain-specific observables (e.g., $\nu_{\max}$, $\Delta\nu$, n_{orders} in the stellar domain).

Kill condition. A pre-specified empirical result that would falsify a specific claim of the framework. All kill conditions were defined before data analysis and none were triggered.

All data are from public sources: IRIS Earth Model Collaboration (seismic tomography models), Kepler/MAST archive (stellar light curves), APOKASC-2 catalog via VizieR (Pinsonneault et al. 2018), ArXiv submission statistics, Hazen et al. 2008 (mineral evolution), published Grand Canyon stratigraphy, ICSD/CAS material species databases, USGS Mineral Commodity Summaries (silicon, lithium, rare earth elements), and Ashby 2013 (engineering materials). All analysis code is available in the corresponding experiment directories. No result has been externally peer-reviewed.